

# X-ray and neutron sources

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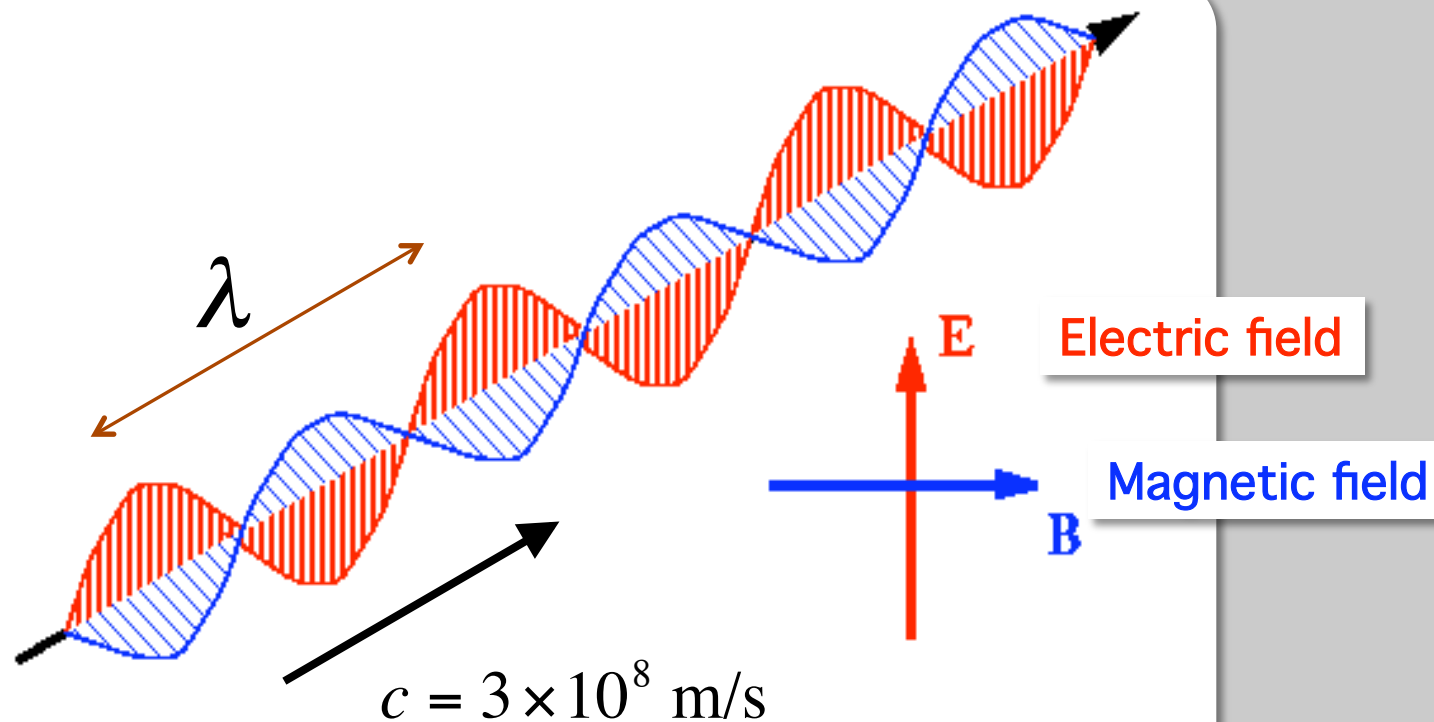
Insubria International Summer School on  
Crystallography for Health and Biosciences  
Como (Italy), June, 18<sup>th</sup> – 23<sup>rd</sup> 2012

# Overview

1. Mechanisms of X-ray production
2. Laboratory X-ray sources
3. Synchrotron Radiation
  - SR sources
  - SR from bending magnets
  - SR from insertion devices
4. Neutron sources

# Mechanisms of X-ray production

# X-rays are electromagnetic radiation



Superposition  
of sinusoidal waves

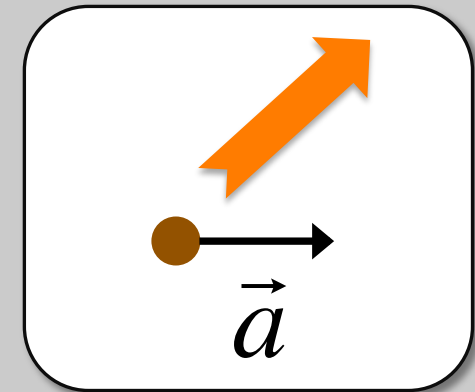
## X-rays

$$\lambda \approx 0.01 \div 10 \text{ \AA}$$
$$\nu \approx 10^{17} \div 10^{20} \text{ Hz}$$
$$E \approx 1 \div 400 \text{ keV}$$

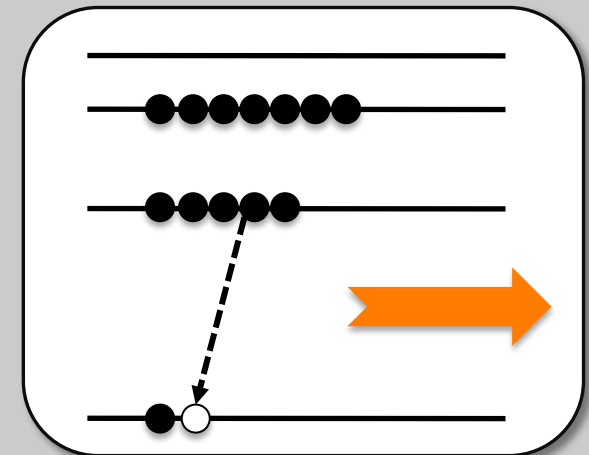
# Two mechanisms ...

## ... of production of electromagnetic radiation

1. Emission from accelerated electric charges

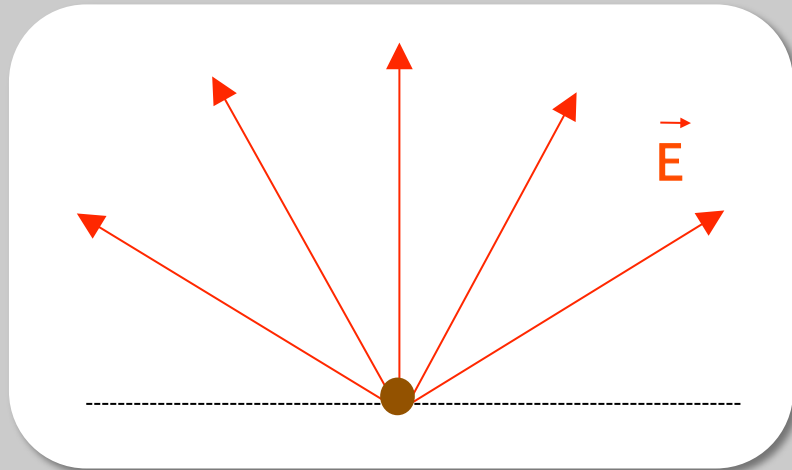


2. Emission as effect of quantum transitions

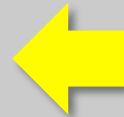


# Non-accelerated charges

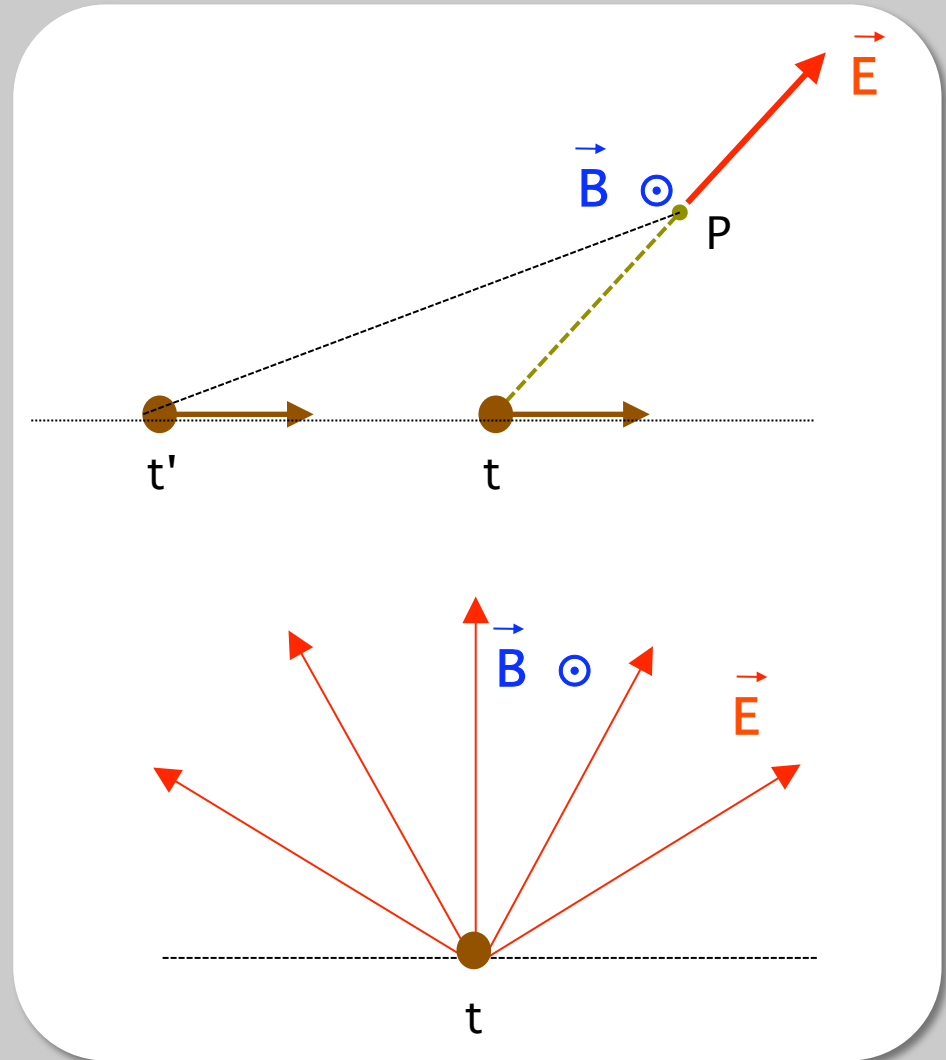
Charge at rest



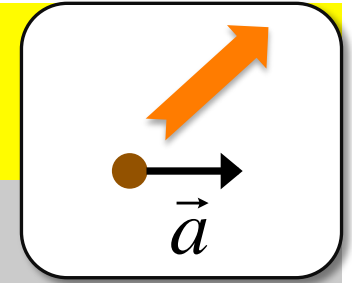
No power is emitted  
as electromagnetic radiation



Constant velocity



# Accelerated charge: a tutorial example (a)



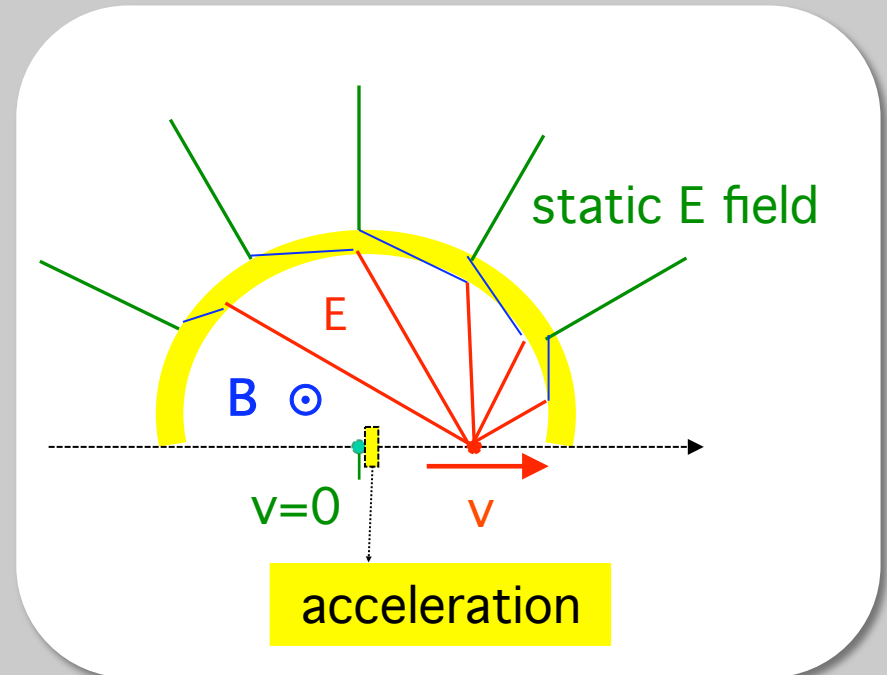
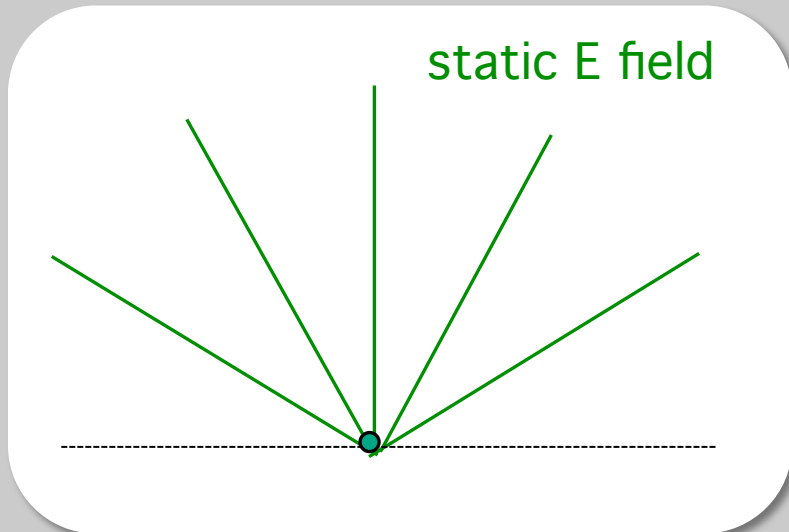
Charge at rest



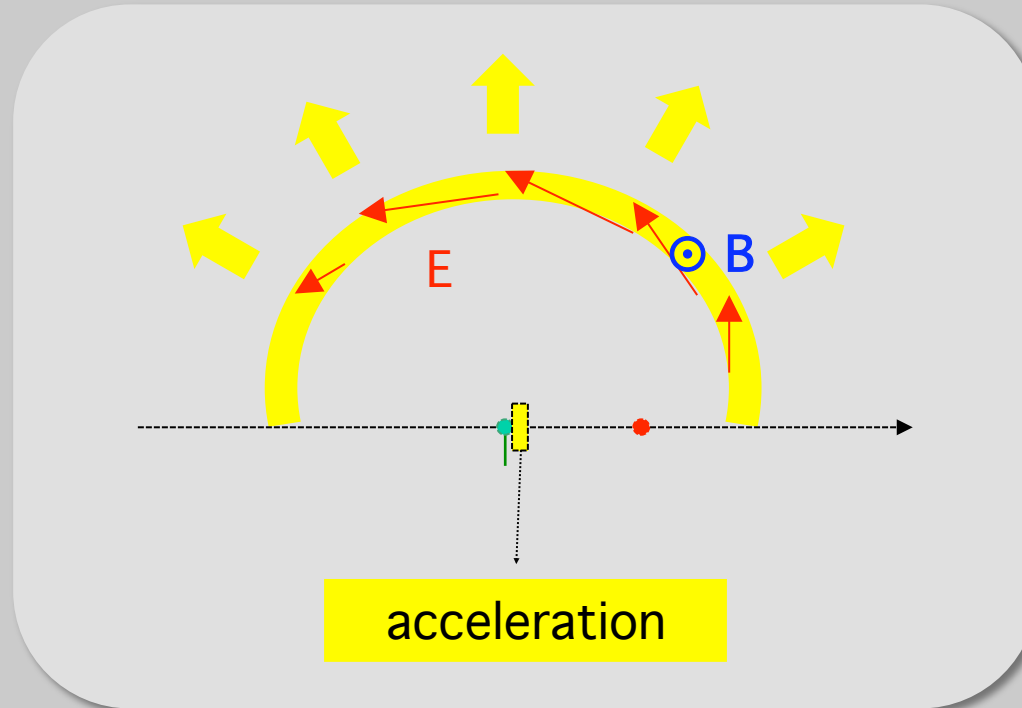
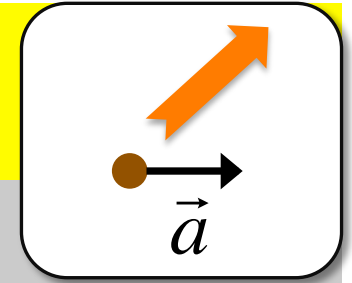
Short acceleration



Constant velocity



# Accelerated charge: a tutorial example (b)



Acceleration

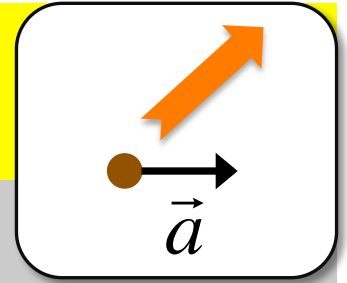


Perturbation  
of static fields

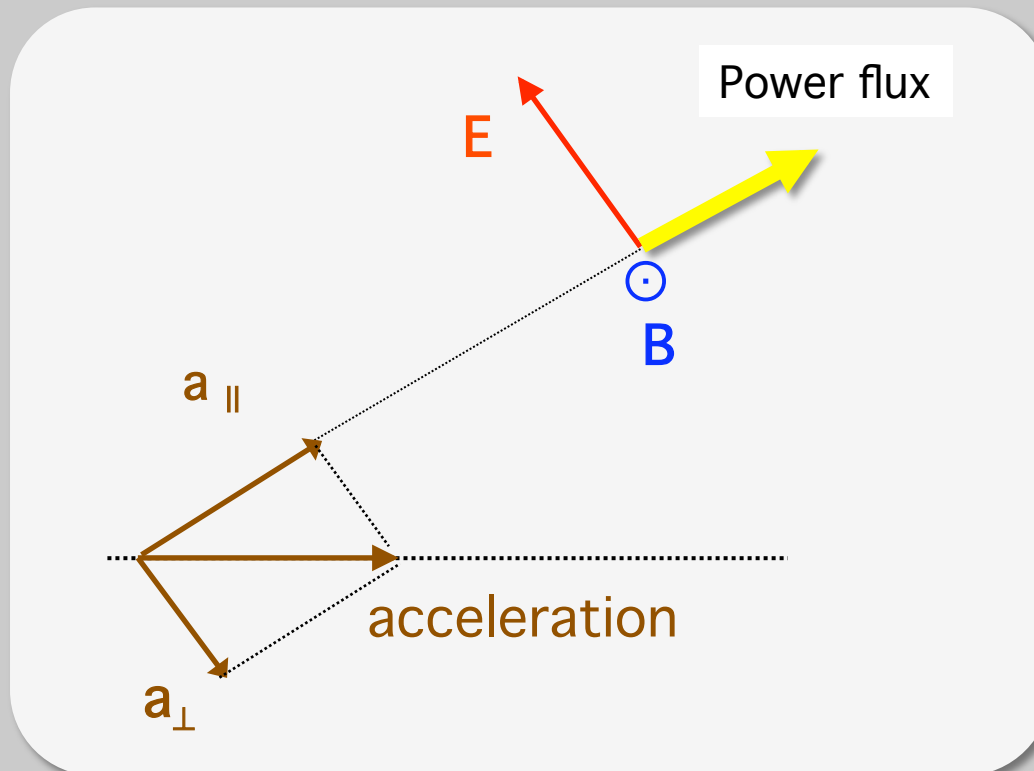
- Electro-magnetic field
- Propagating with velocity  $c = 3 \times 10^8$  m/s
- Carrying energy



# Dipole approximation



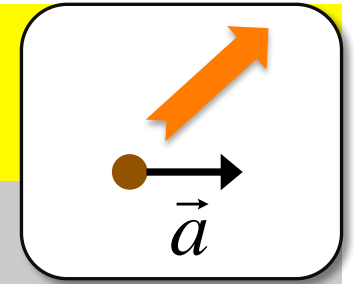
- charge velocity:  $v \ll c$
- charge size:  $d \ll \lambda$
- observer distance:  $r \gg \lambda$



$$\vec{E}(\vec{r}, t) = -\frac{q\vec{a}_{\perp}(t')}{4\pi\epsilon_0 r c^2}$$

$$\vec{B}(\vec{r}, t) = \frac{\hat{r} \times \vec{E}(\vec{r}, t)}{c}$$

# Dipole approximation: radiated power

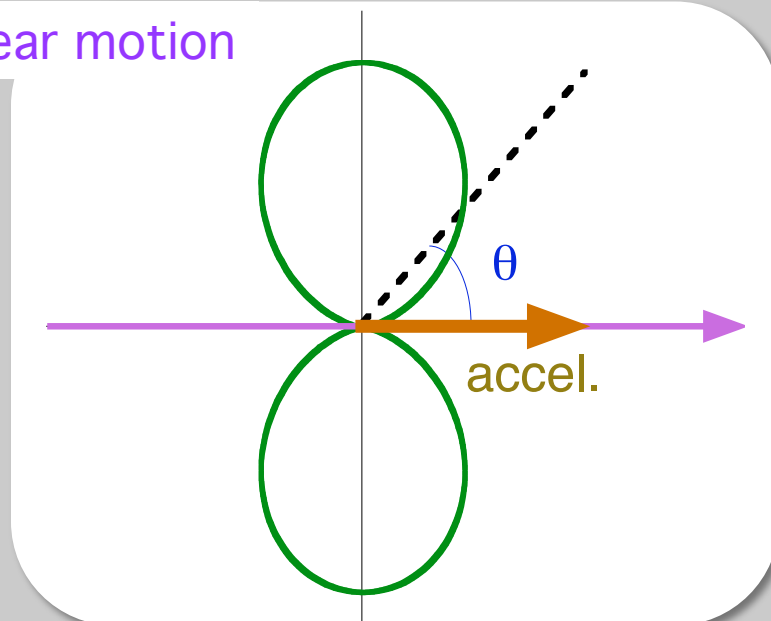


The emitted power is:

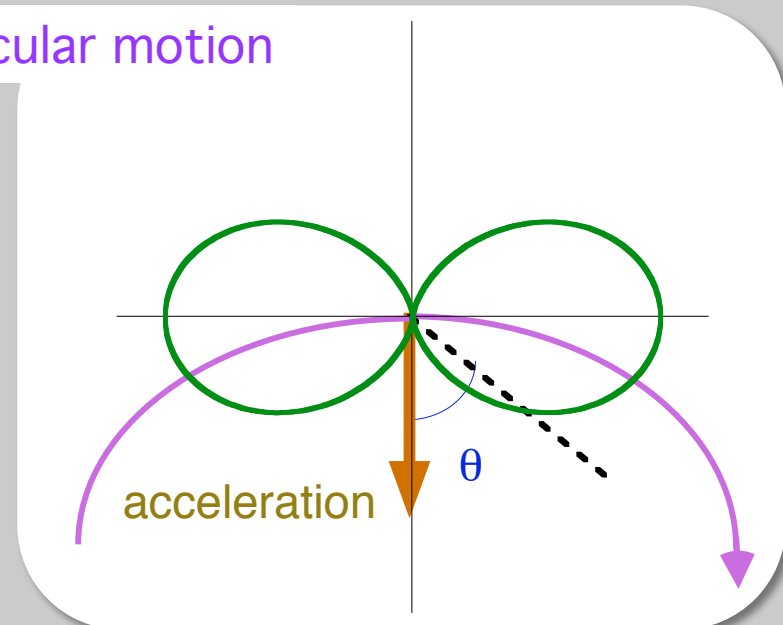
- Zero in the direction of acceleration
- Maximum in the perpendicular plane

## Angular emission profiles

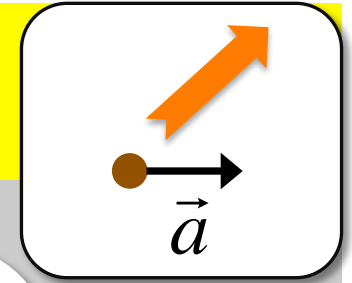
Linear motion



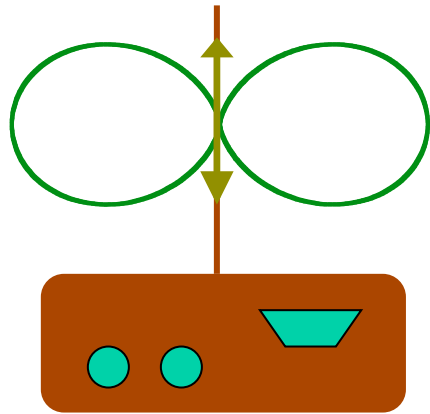
Circular motion



# Examples

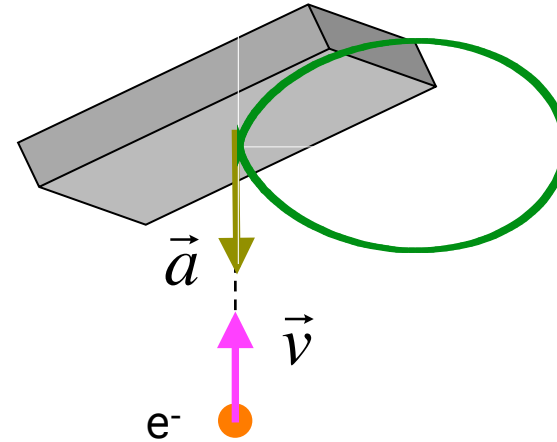


## Oscillating motion



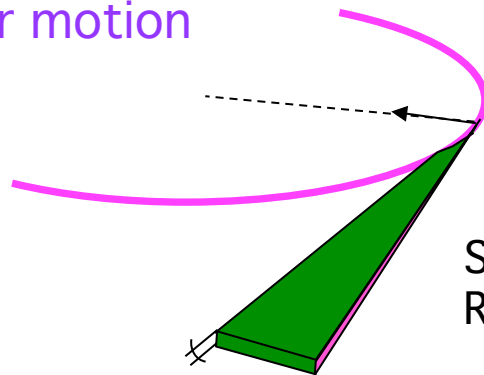
Broadcasting antenna

## Average linear deceleration



Bremsstrahlung  
in X-ray tubes

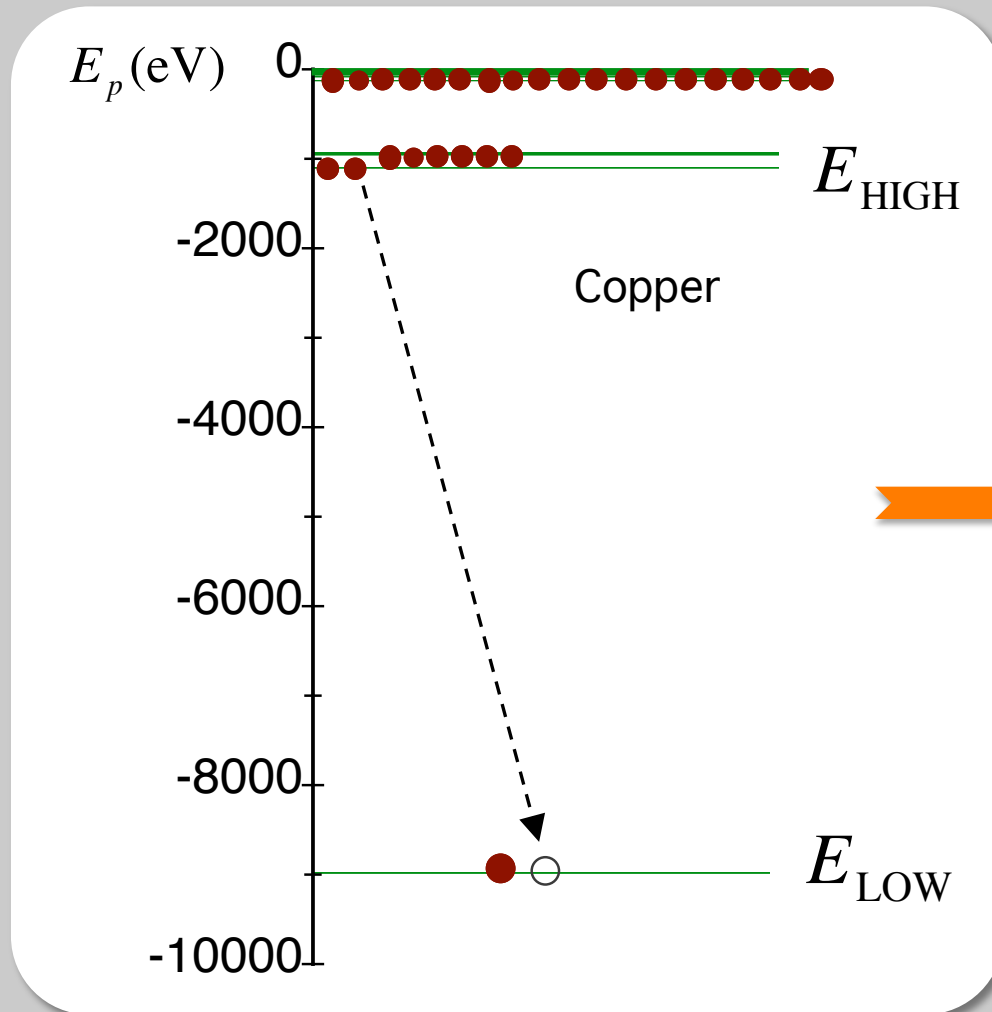
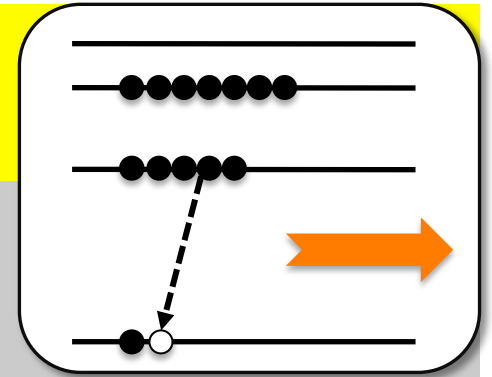
## Relativistic circular motion



Synchrotron  
Radiation

No dipole approximation  
Relativistic treatment necessary

# Quantum transitions

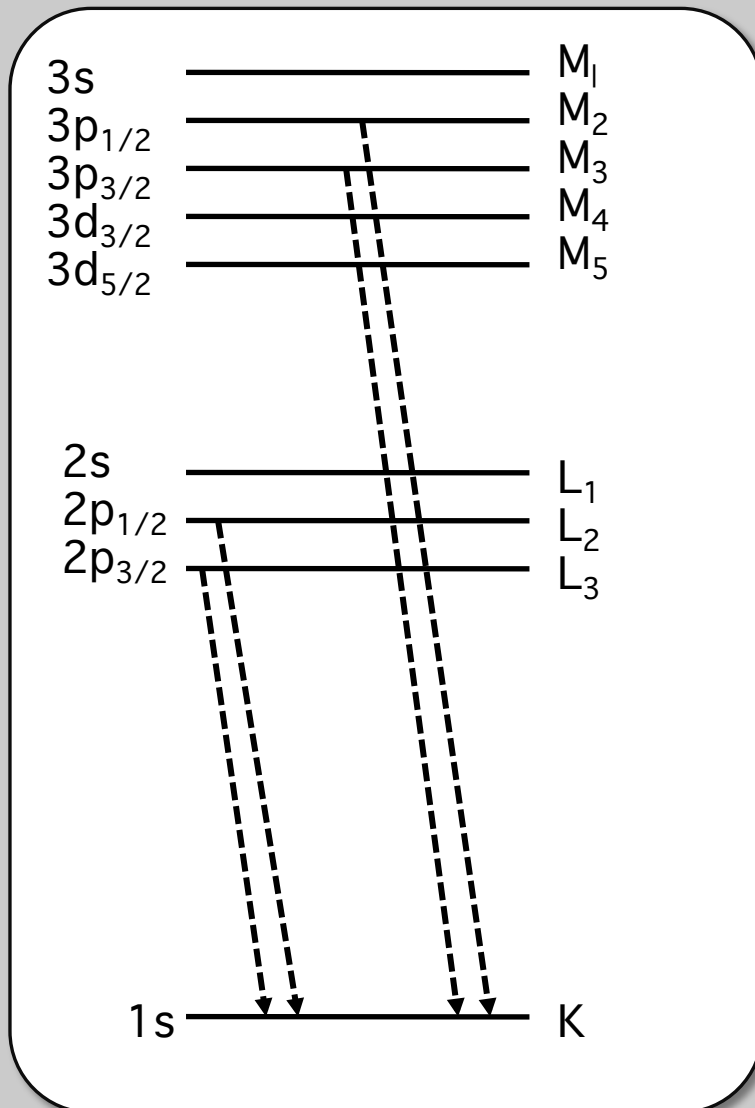
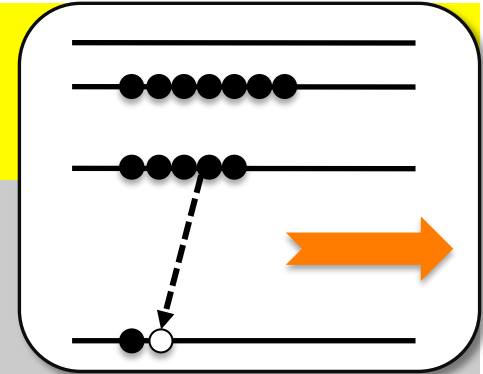


$$\hbar\omega = E_{\text{HIGH}} - E_{\text{LOW}}$$

Atomic X-ray transitions

$$\hbar\omega \approx 0.4 \div 100 \text{ keV}$$

# Characteristic lines



## Transitions and X-ray lines

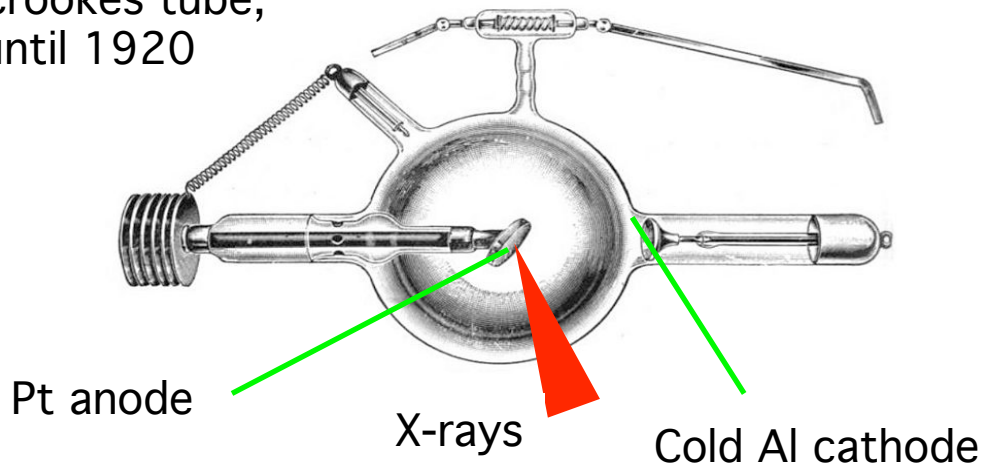
K - L <sub>3</sub>	Kα <sub>1</sub>	Kα
K - L <sub>2</sub>	Kα <sub>2</sub>	
K - M <sub>3</sub>	Kβ <sub>1</sub>	Kβ
K - M <sub>2</sub>	Kβ <sub>3</sub>	

(Electric-dipole allowed transitions)

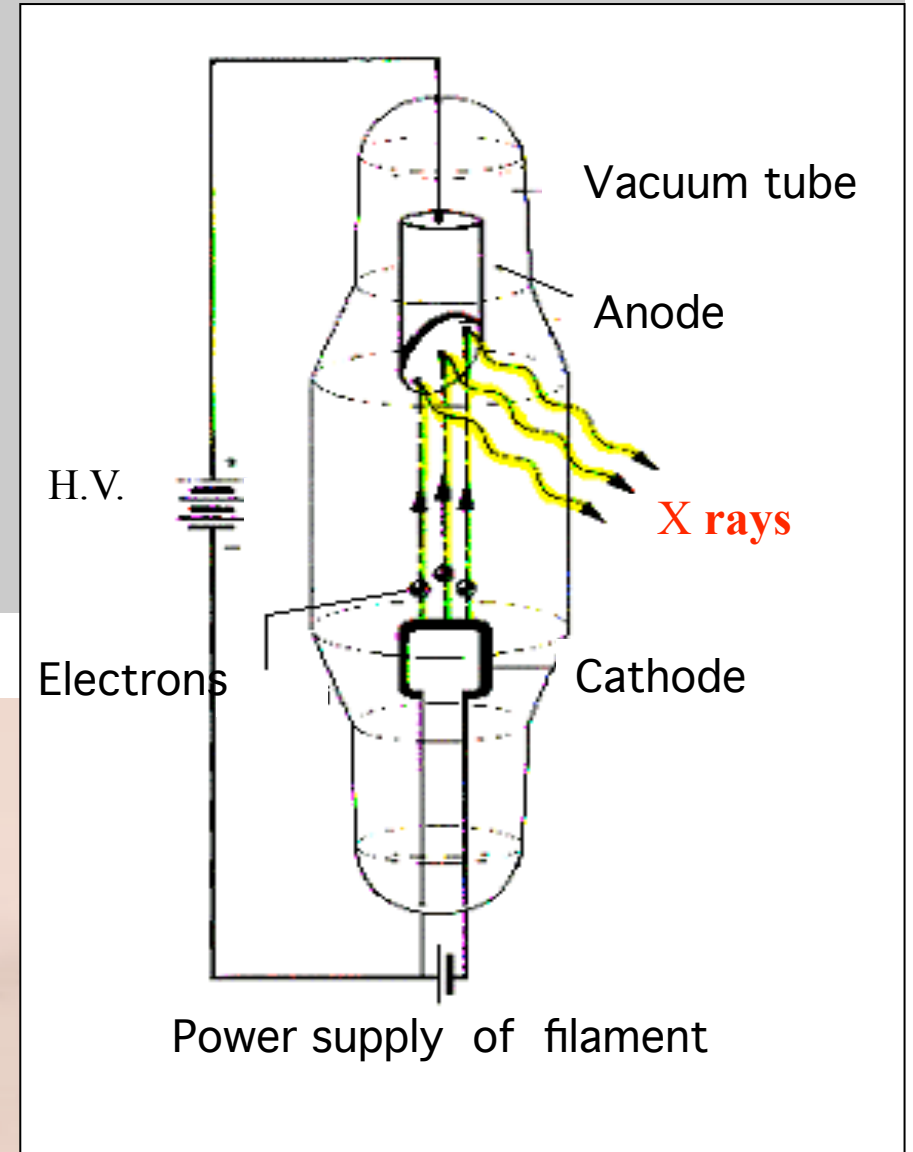
# Laboratory X-ray sources

# X-ray tubes

Crookes tube,  
until 1920



Hot cathode Coolidge's tube (1913)



# Present-day laboratory tubes



High vacuum sealed tubes

Independent control of I and V

Accelerating voltage < 100 kV

Different anode metals:  
Cr, Cu, Mo, W, ...

Power: 1 - 4 kW

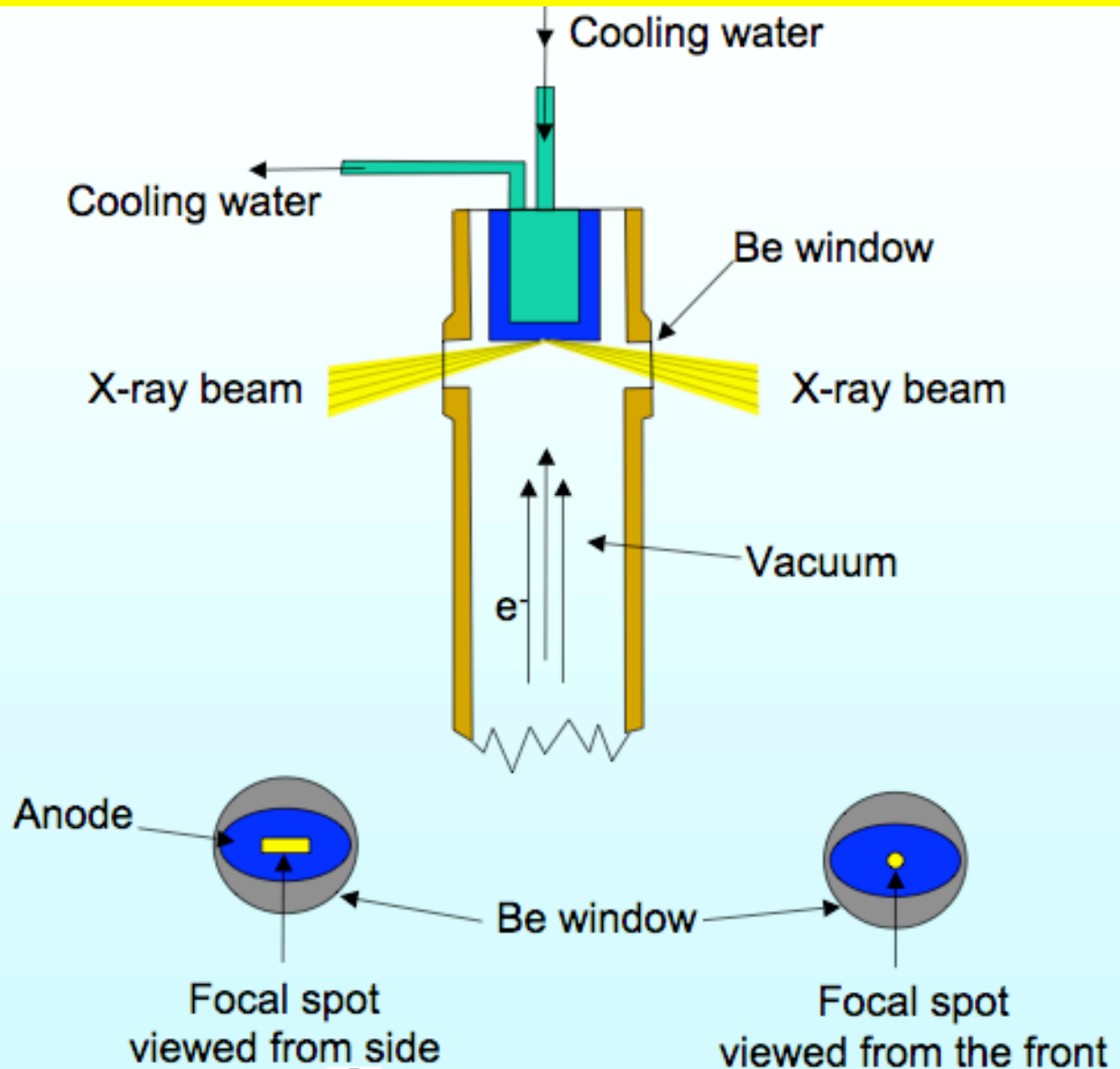
~ 1 % X-ray production

~99 % anode heating

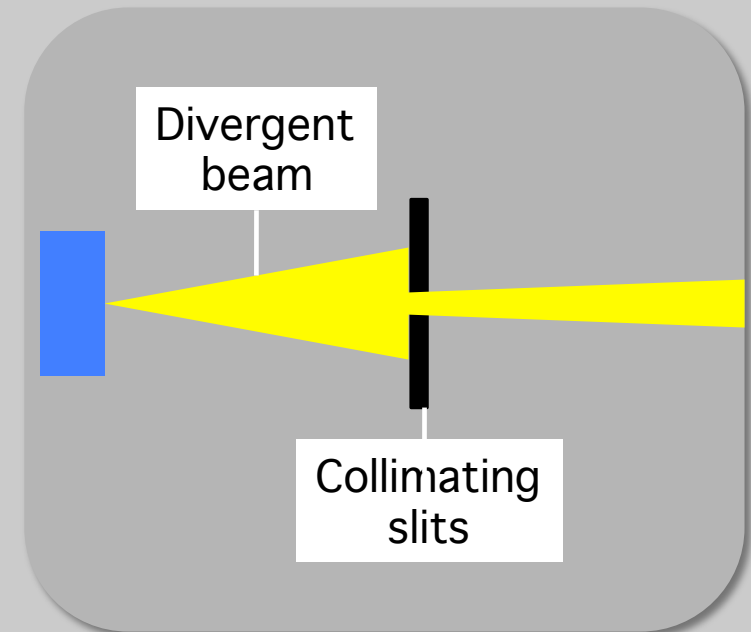
→ water cooling



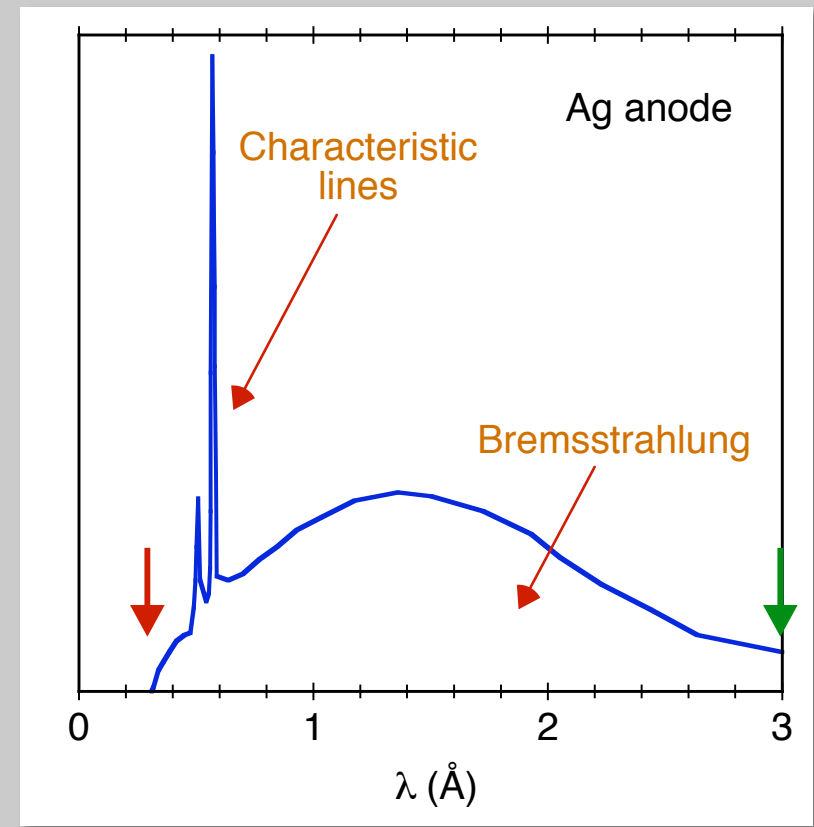
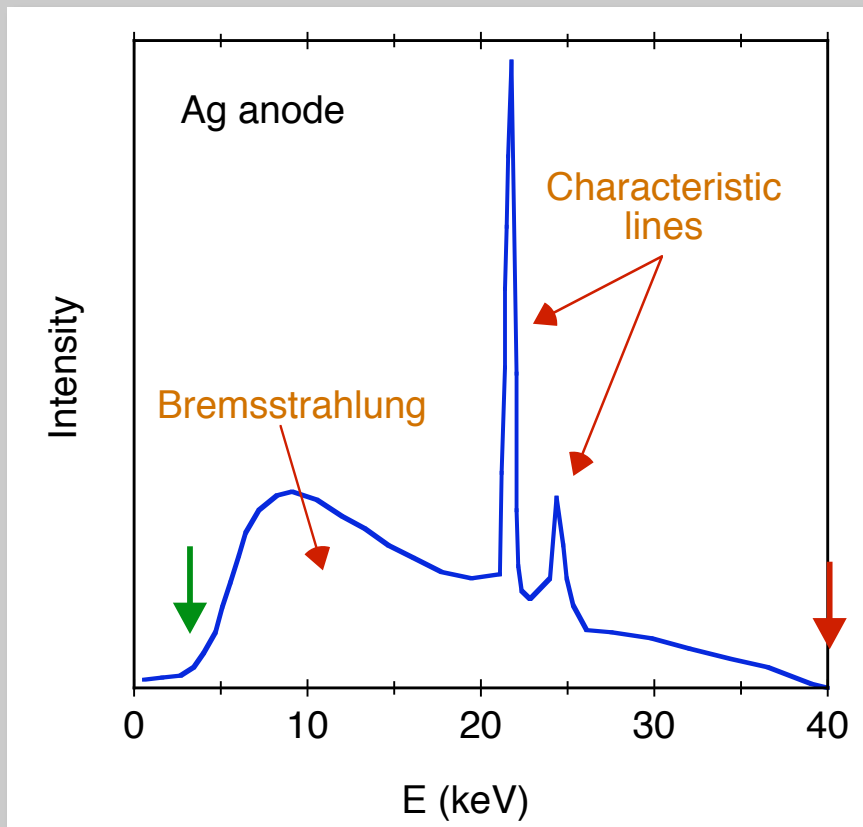
# X-ray emission



Un-polarized X-rays

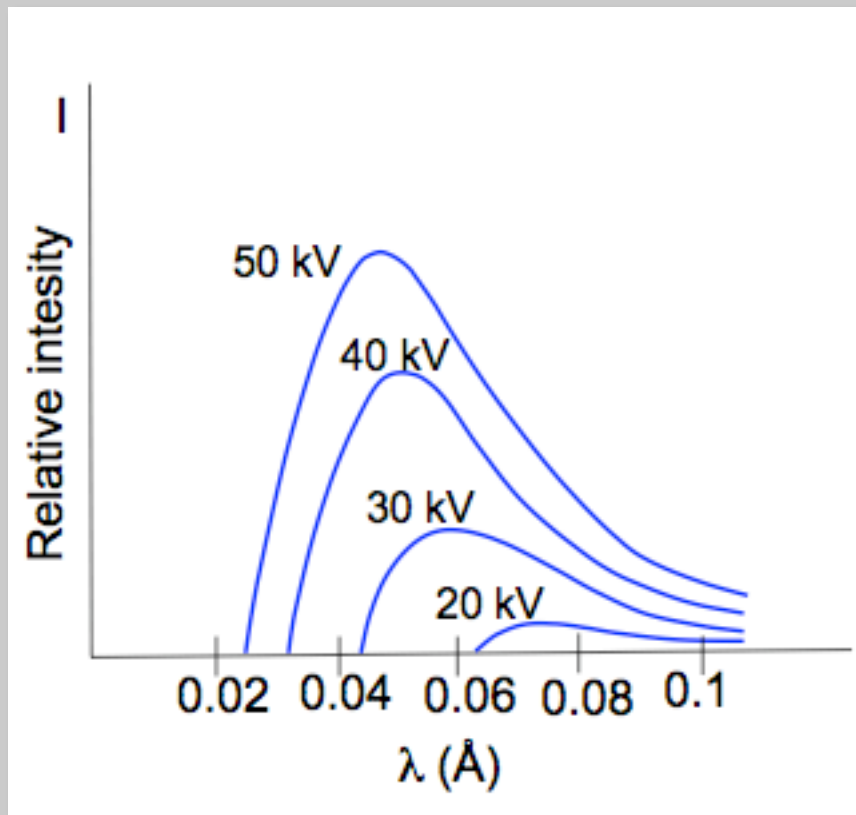


# Emission spectrum



$$\lambda [\text{Å}] = \frac{12.4}{E [\text{keV}]}$$

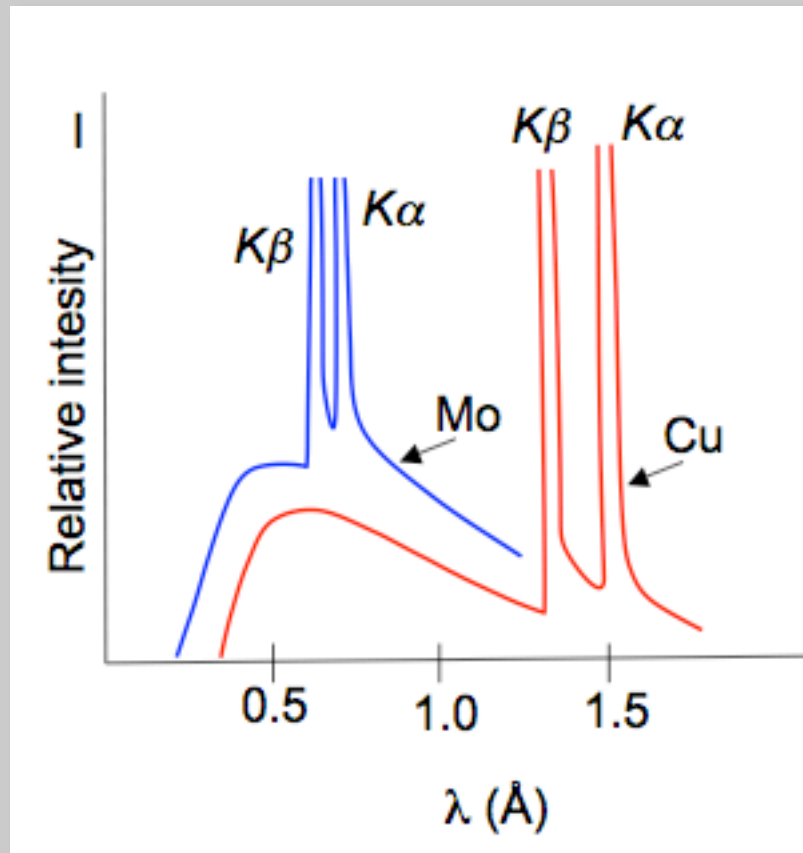
# Continuous bremsstrahlung spectrum



Electrons accelerating voltage:

- Maximum energy (minimum  $\lambda$ )
- Total intensity

# Characteristic lines



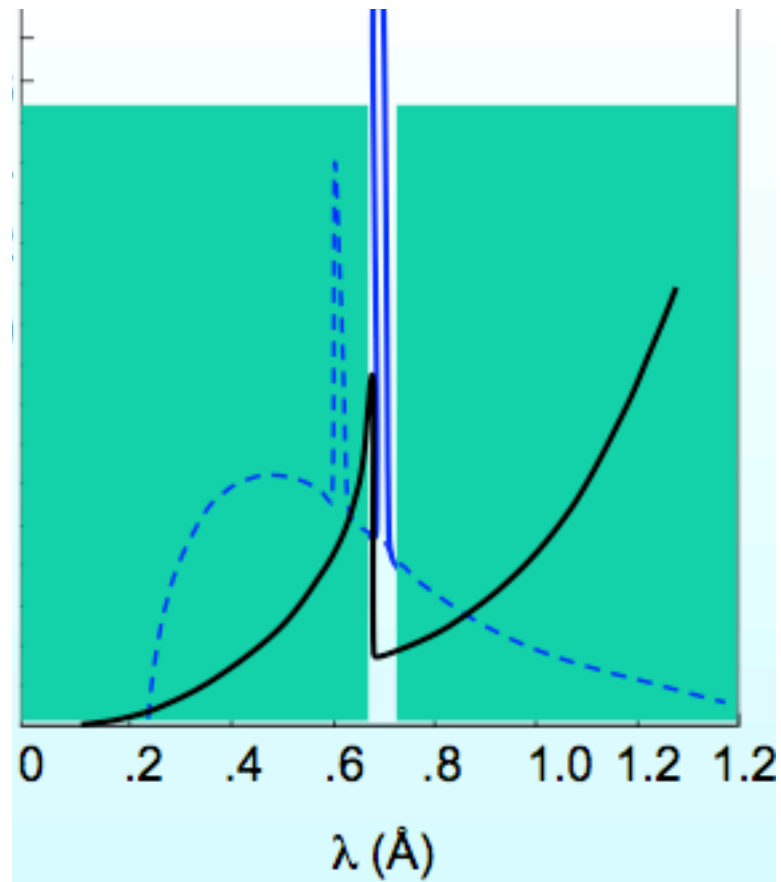
Moseley law

$$\lambda \propto \frac{1}{Z^2}$$

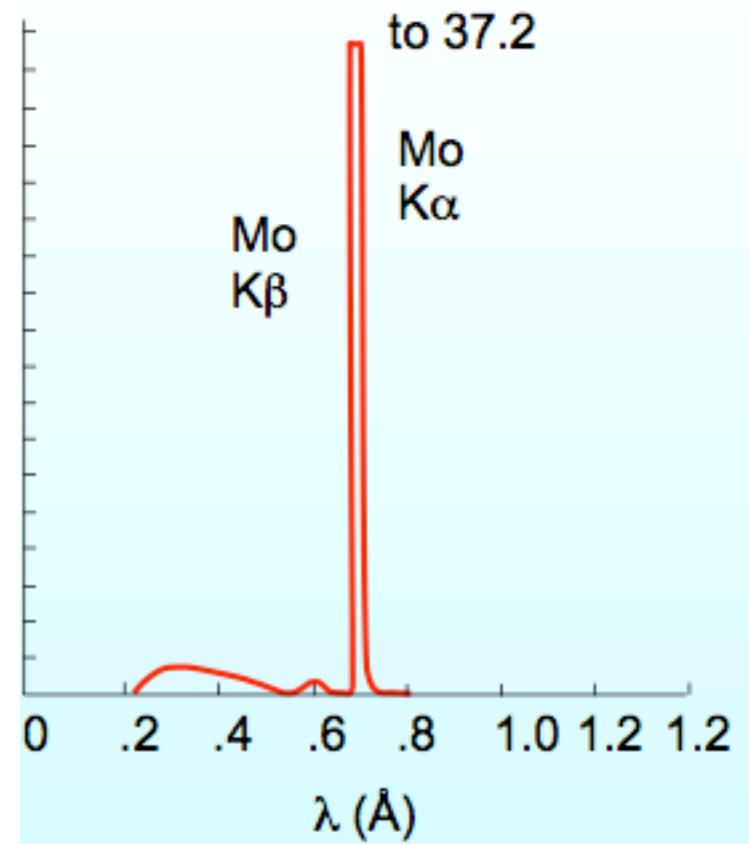
		Cr	Cu	Mo
$K\alpha$	$K\alpha_1$	2.2897	1.5406	0.7093
	$K\alpha_2$	2.2936	1.5444	0.7136
$K\beta$		2.0849	1.3922	0.6323

# Filters

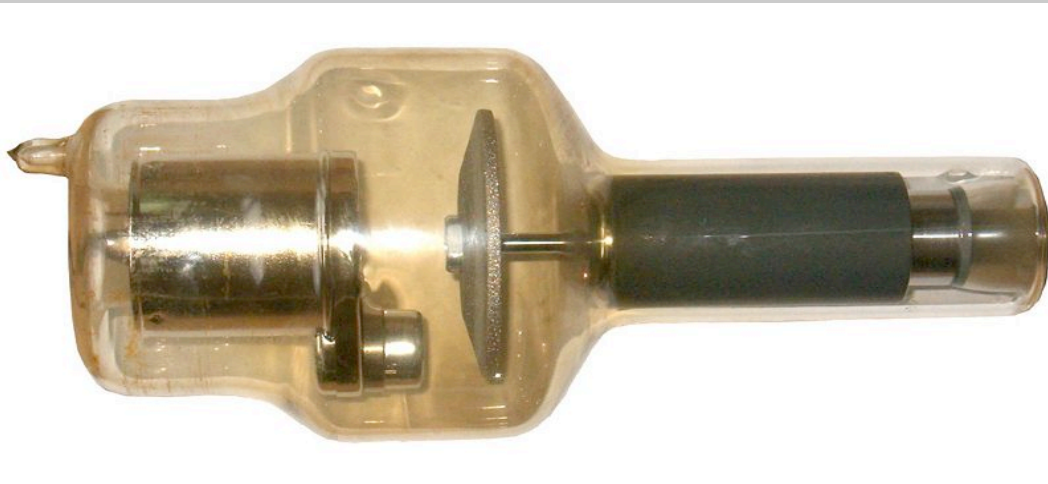
- Mo emission spectrum
- Zr absorption coefficient



- Mo filtered spectrum

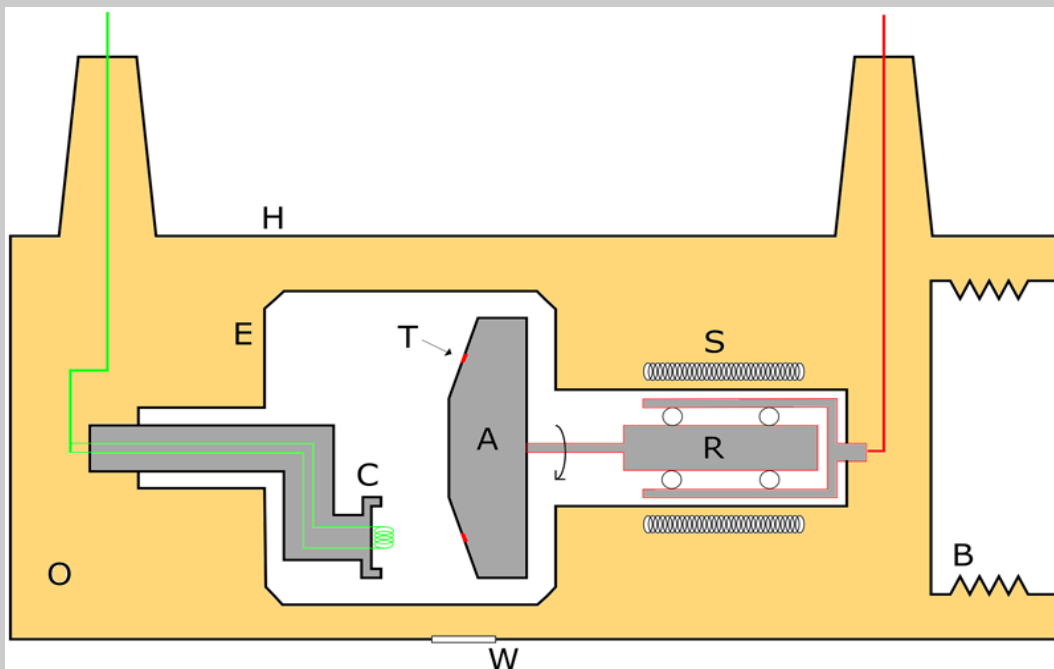


# Rotating-anode tubes



Snapshot emission:  
no liquid cooling

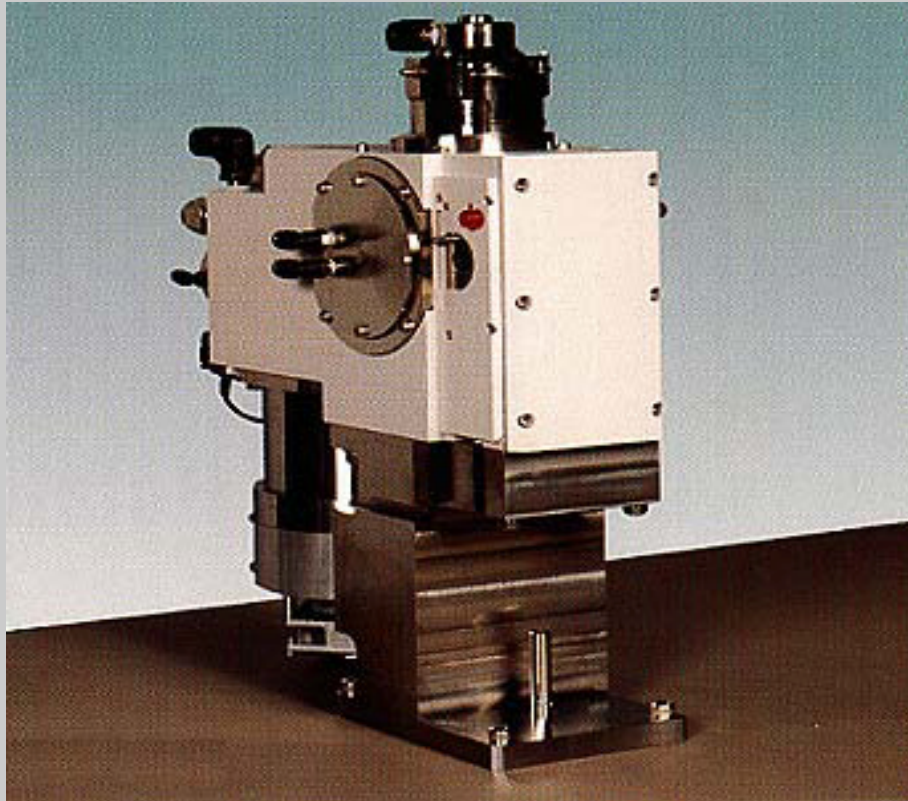
Power ratings up to 100 kW



Anode rotated by  
electromagnetic induction

Almost all medical tubes  
are rotating anode tubes  
(exception: dental tubes)

# Liquid cooled rotating-anode sources



Power ratings 12 – 18 kW

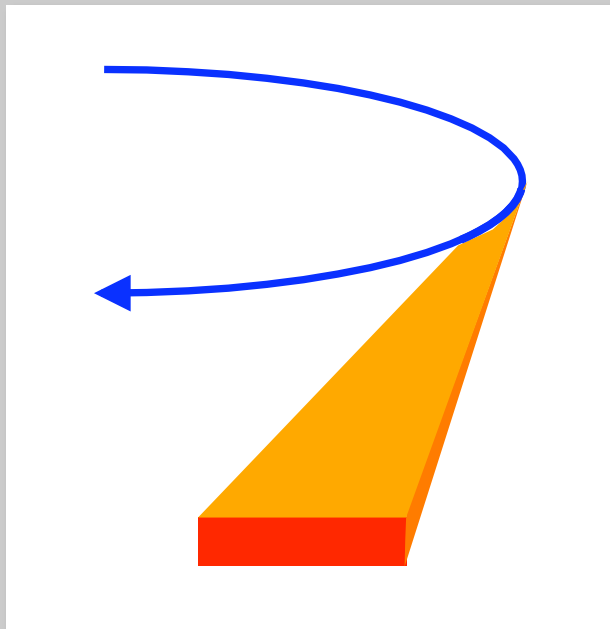
For non-snapshot applications: diffraction, tomography, etc.

# Synchrotron Radiation



# Synchrotron radiation

Electromagnetic radiation emitted by centripetally accelerated electrons moving at relativistic speed



Storage rings

Relative velocity

$$\beta = \frac{v}{c} \approx 1$$

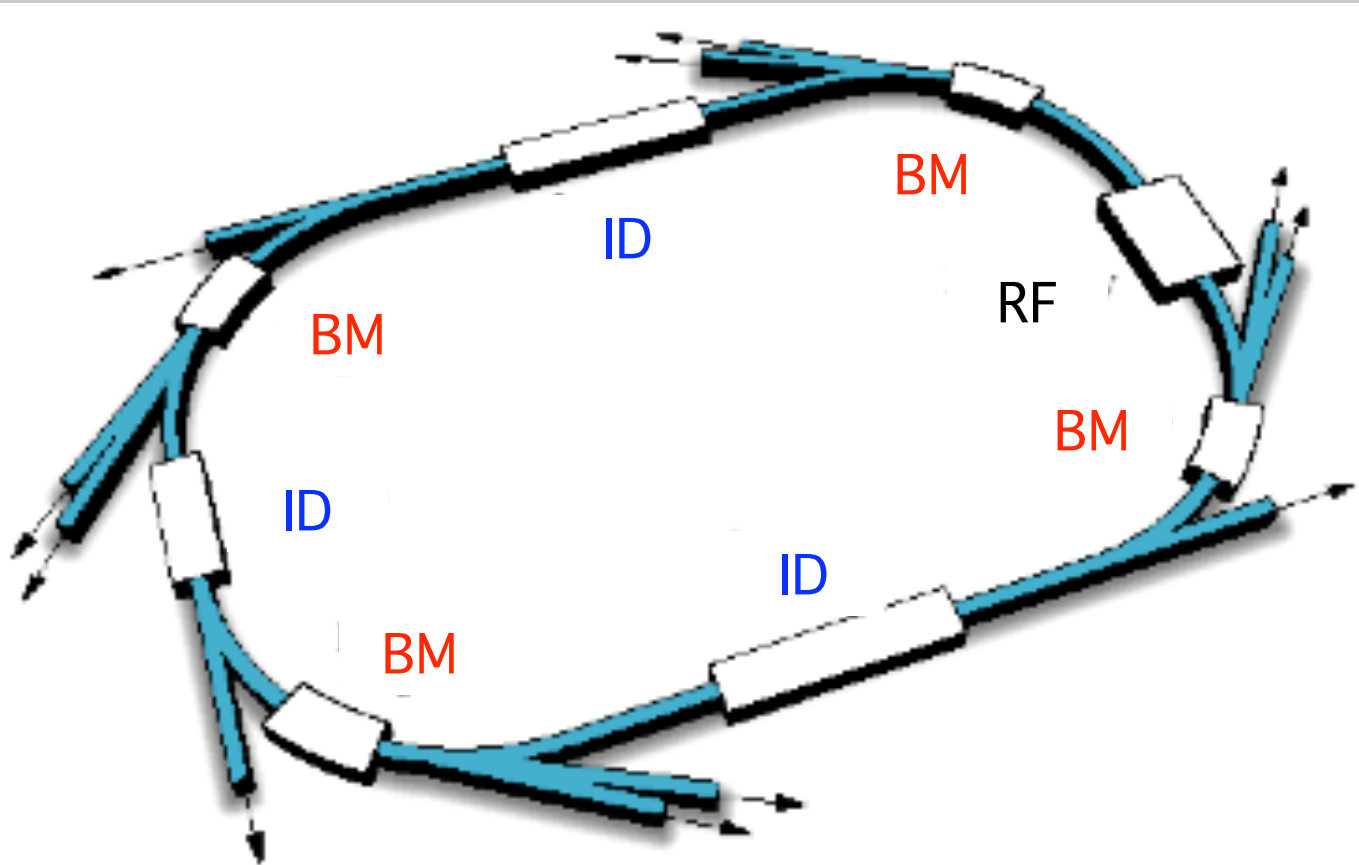
Electron energy

$$W \cong 1 \div 10 \text{ GeV}$$

A key parameter

$$\begin{aligned} \gamma &= \left(1 - v^2 / c^2\right)^{-1/2} \\ &= W / m_0 c^2 \\ &\approx 2000 \div 20000 \end{aligned}$$

# Storage rings as S.R. sources



## Basic components

BM = bending magnets

ID = insertion devices

RF = radiofrequency cavity

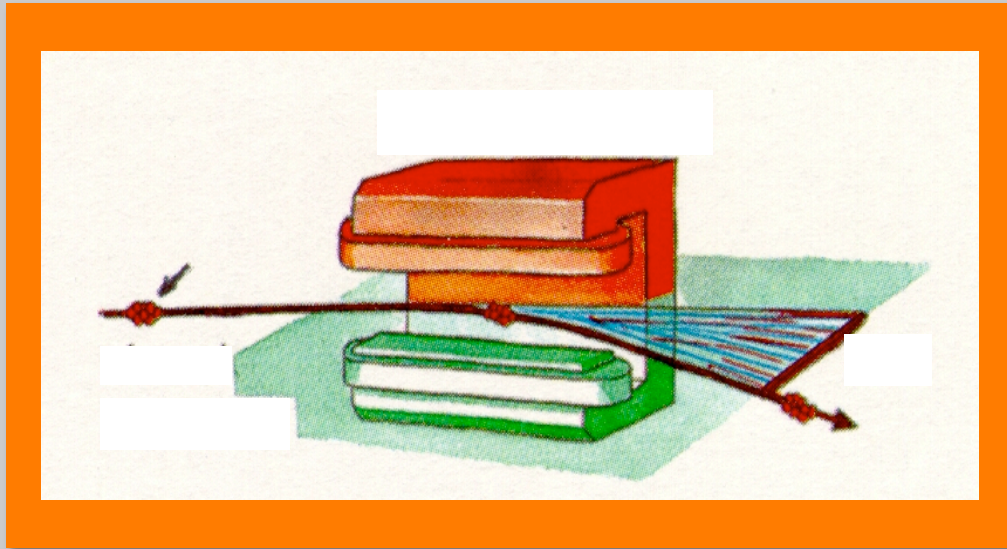
Synchrotron Radiation from:

➤ **Bending magnets**

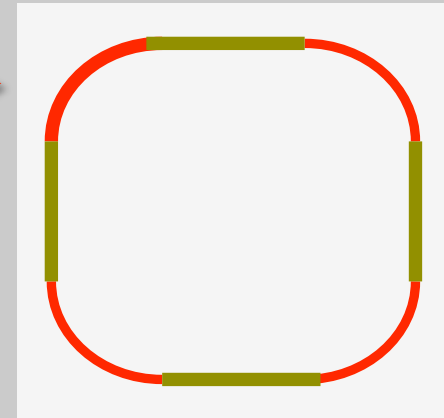
➤ **Insertion devices ...**

- wigglers
- undulators

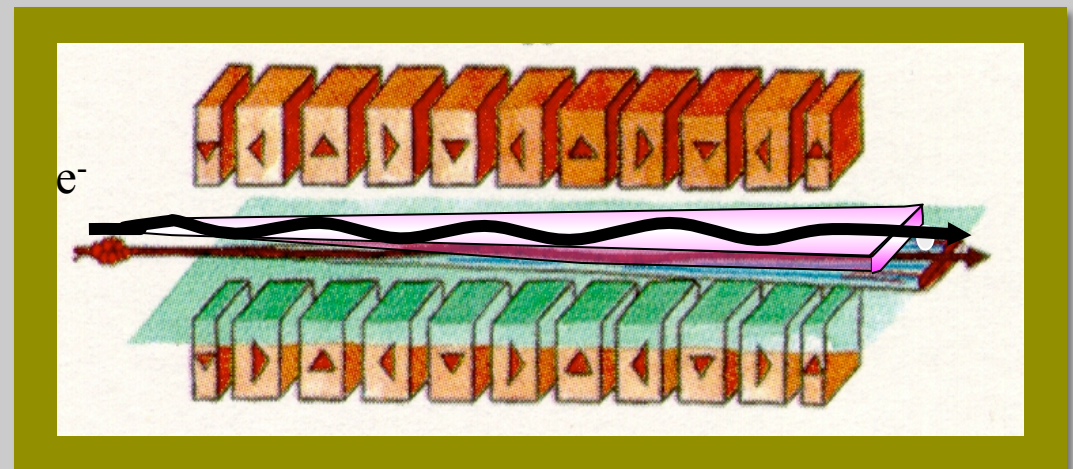
# Bending magnets and insertion devices



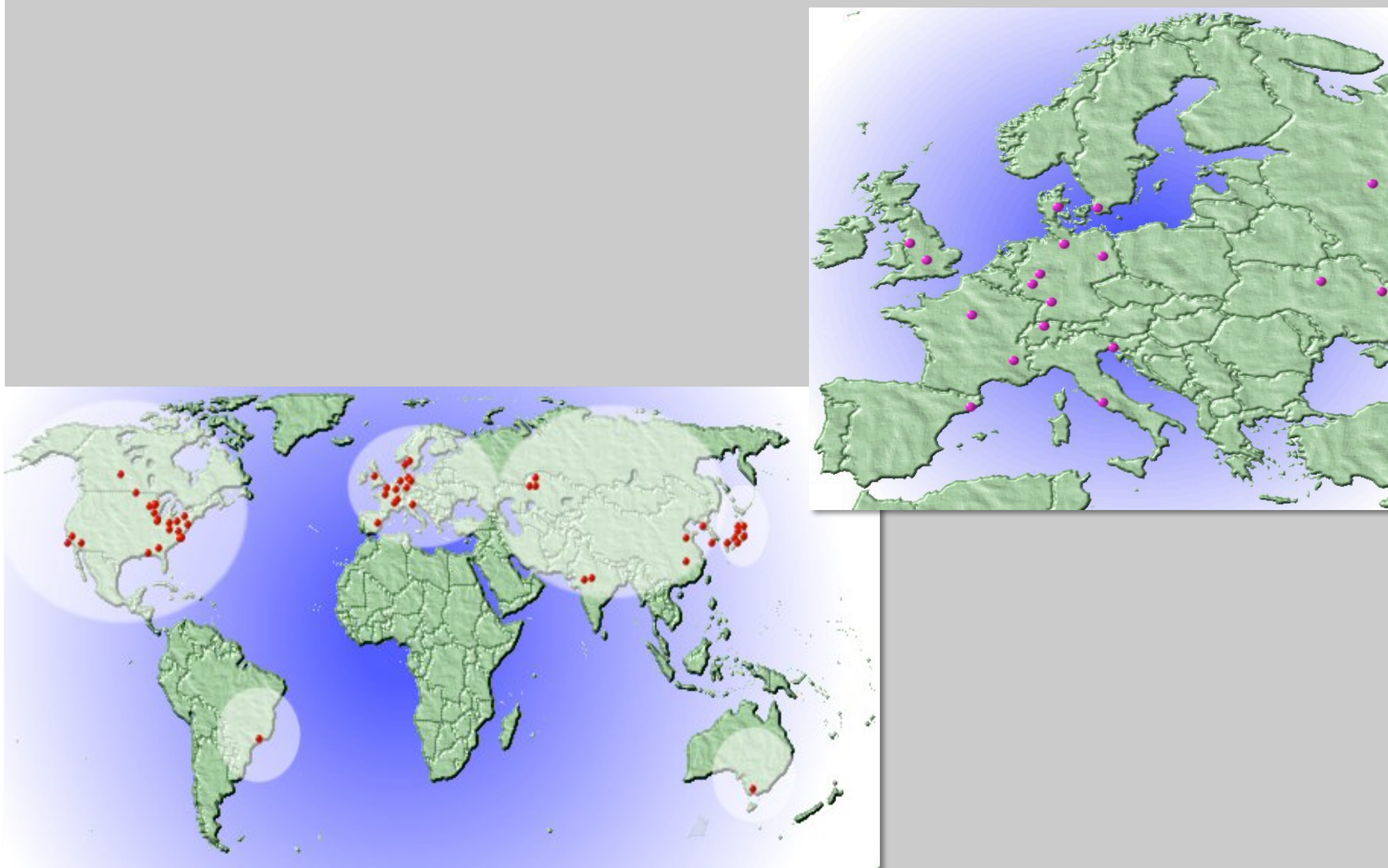
Bending magnet



Insertion device



# S.R. Facilities - map



# S.R. Facilities

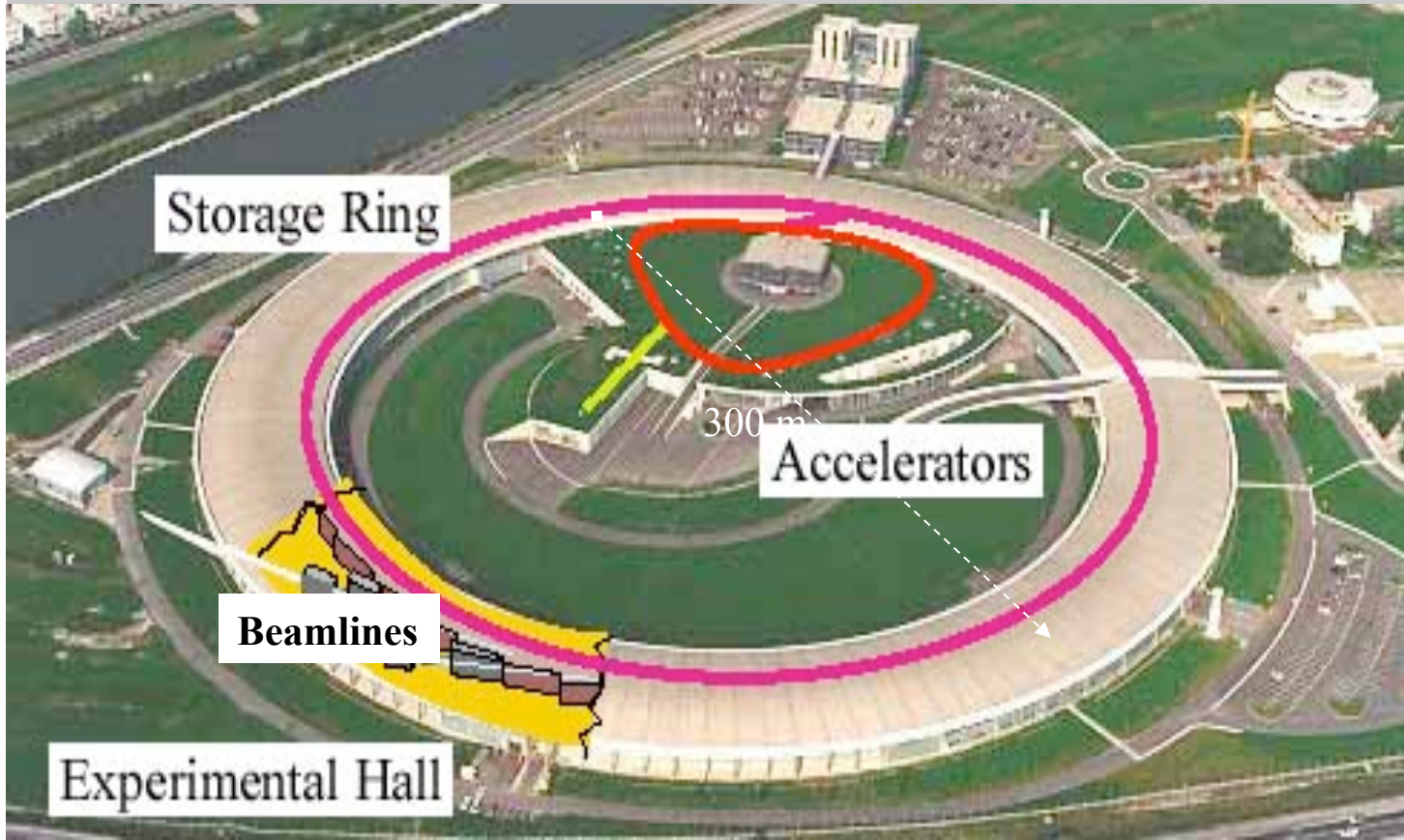
Name	Site	Year	E (GeV)
<b>SPring8</b> Super Photon ring 8 GeV	Hyogo (Japan)	1997	8
<b>APS</b> Advanced Photon Source	Argonne, IL (USA)	1996	7
<b>ESRF</b> European S. R. Facility	Grenoble (France)	1994	6



Name	Site	Year	E (GeV)
<b>PETRA III</b>	Hamburg (D)	2009	6
<b>DORIS III</b>	Hamburg (D)	1980	4.45
<b>Diamond</b>	Didcot (UK)	2007	3
<b>Soleil</b>	S.Aubin (F)	2006	2.75
<b>Elettra</b>	Trieste (I)	1994	2.4
<b>ALBA</b>	Barcelona (E)	2012	3.0
<b>SLS</b>	Villigen (CH)	2001	2.4



# E.S.R.F = European Synchrotron Radiation Facility



Electron energy

$$W = 6 \text{ GeV}$$

$$\gamma = 12000$$

Diameter

300 m

32 bending magnets  
32 straight sections

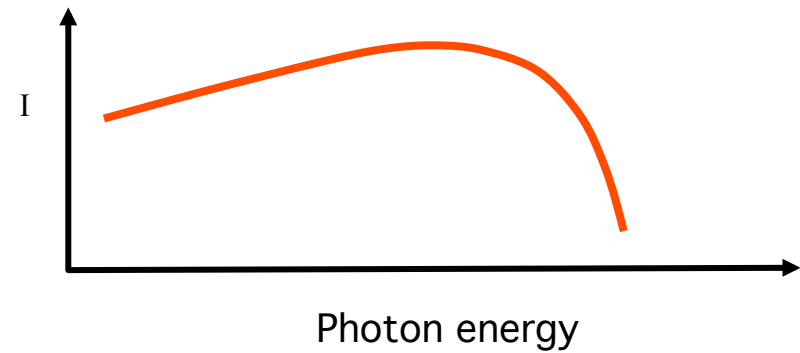
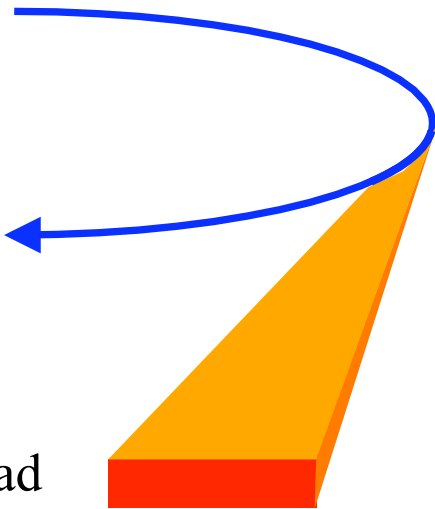
15 BM beamlines  
32 ID beamlines

**S. R. from bending magnets**

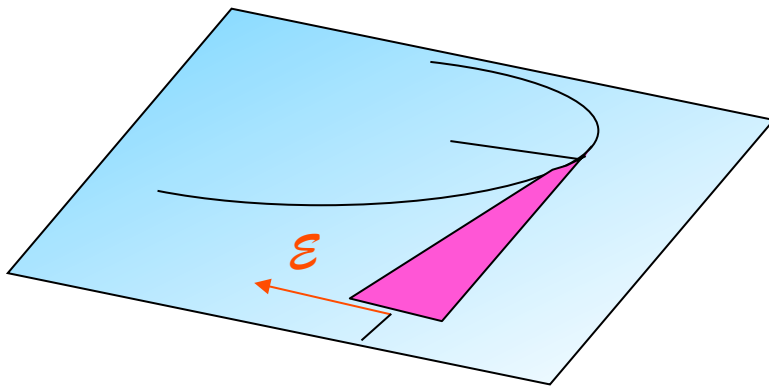
# Properties of Synchrotron Radiation

Collimation

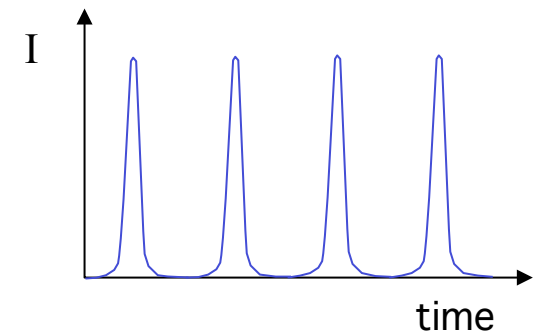
$$\Theta \cong 1/\gamma \text{ rad}$$



High intensity  
continuous spectrum



Polarisation

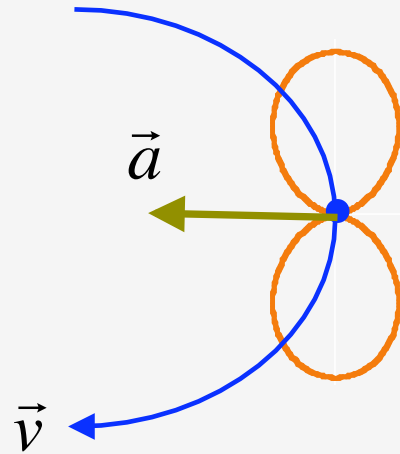


Time structure

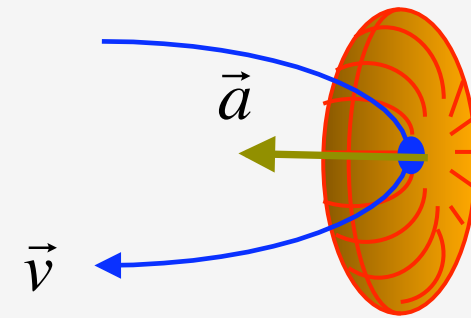


# S.R. angular distribution (a)

$$v \ll c$$



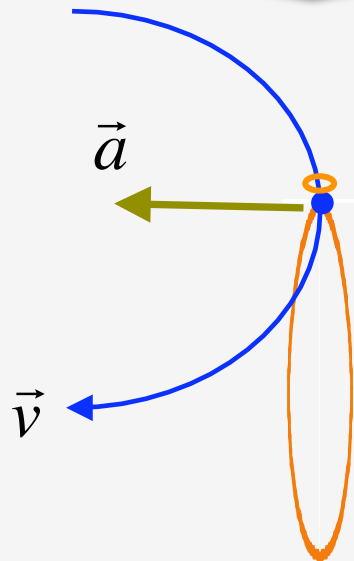
Classical dipole emission pattern



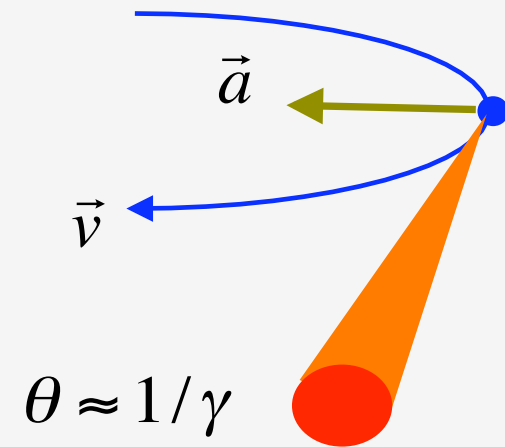
Top view

Perspective

$$v \approx c$$



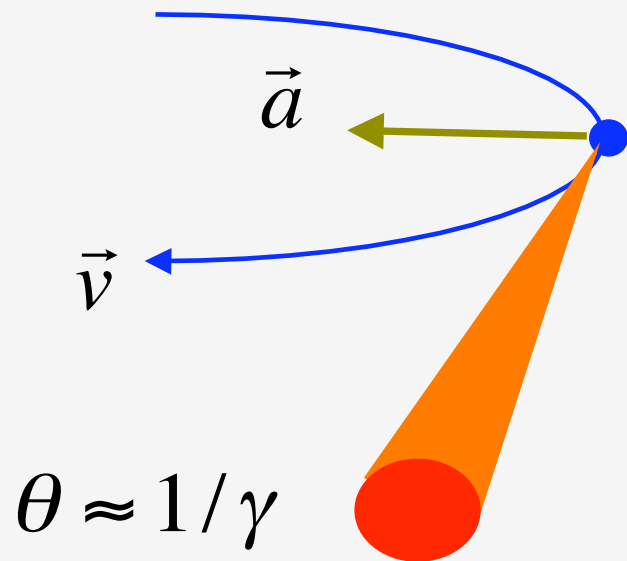
Relativistic emission pattern



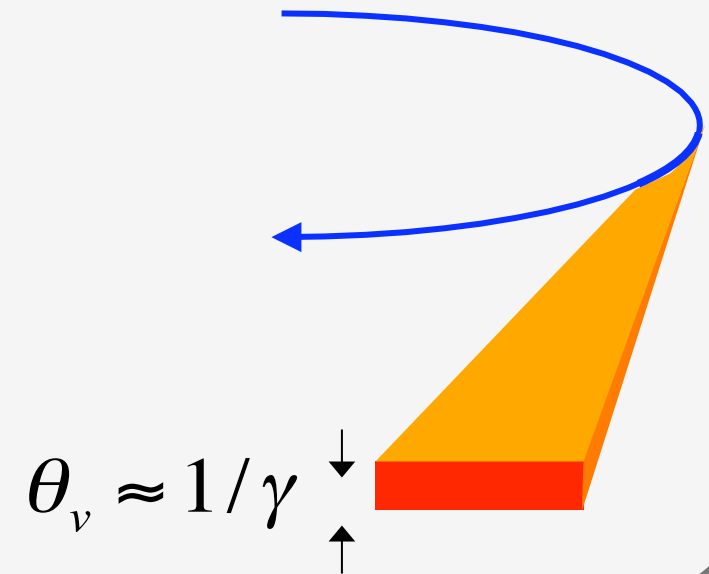
$$\theta \approx 1/\gamma$$

# S.R. angular distribution (b)

Instantaneous emission  
from one electron



Electron beam  
in bending magnet



ESRF:

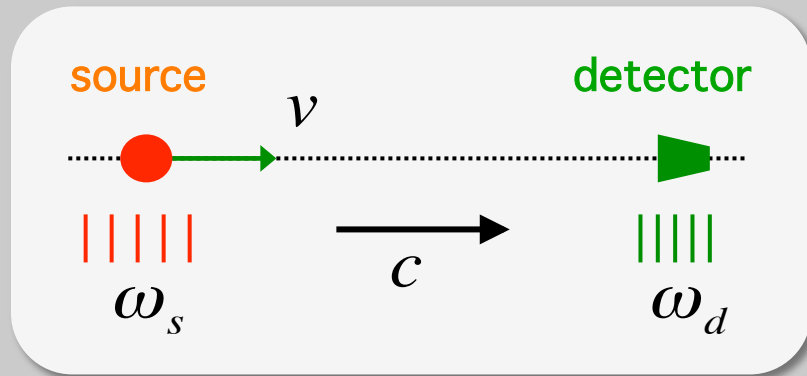
$$W = 6 \text{ GeV}$$

$$\gamma = \frac{W}{m_0 c^2} \approx 12000$$



$$\theta \approx \frac{1}{\gamma} \approx 10^{-4} \text{ rad} \approx 0.005^\circ$$

# Relativistic Doppler effect



$v$  = approaching velocity  
(detector .vs. source)

$c$  = electrom. wave velocity  
(indep. of reference)

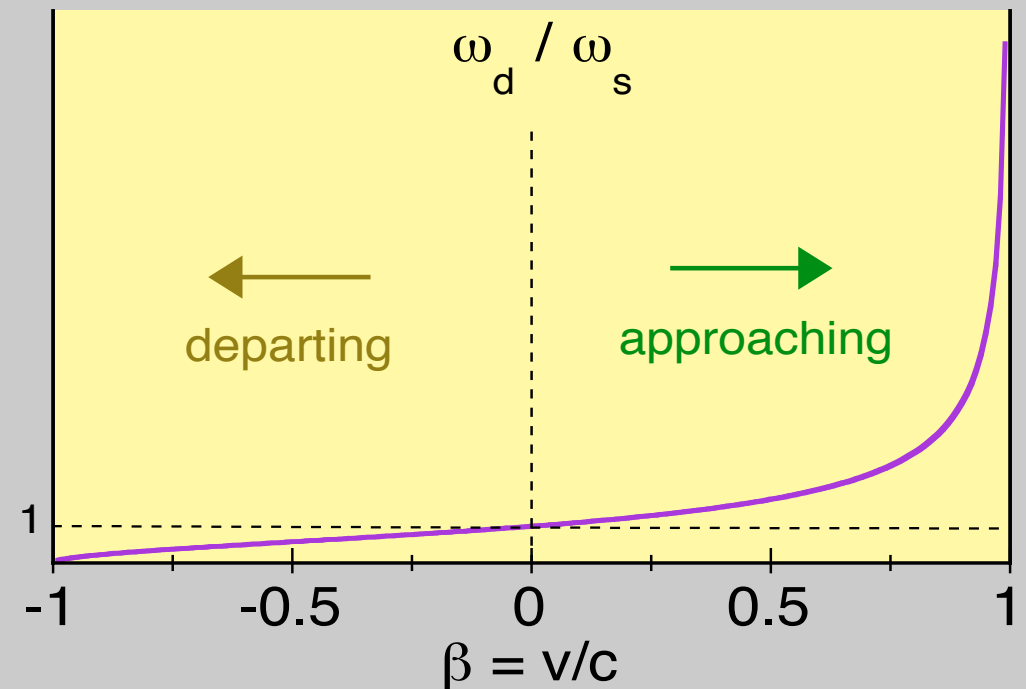
Lorentz-invariance

$$kx - \omega t = k' x' - \omega' t'$$

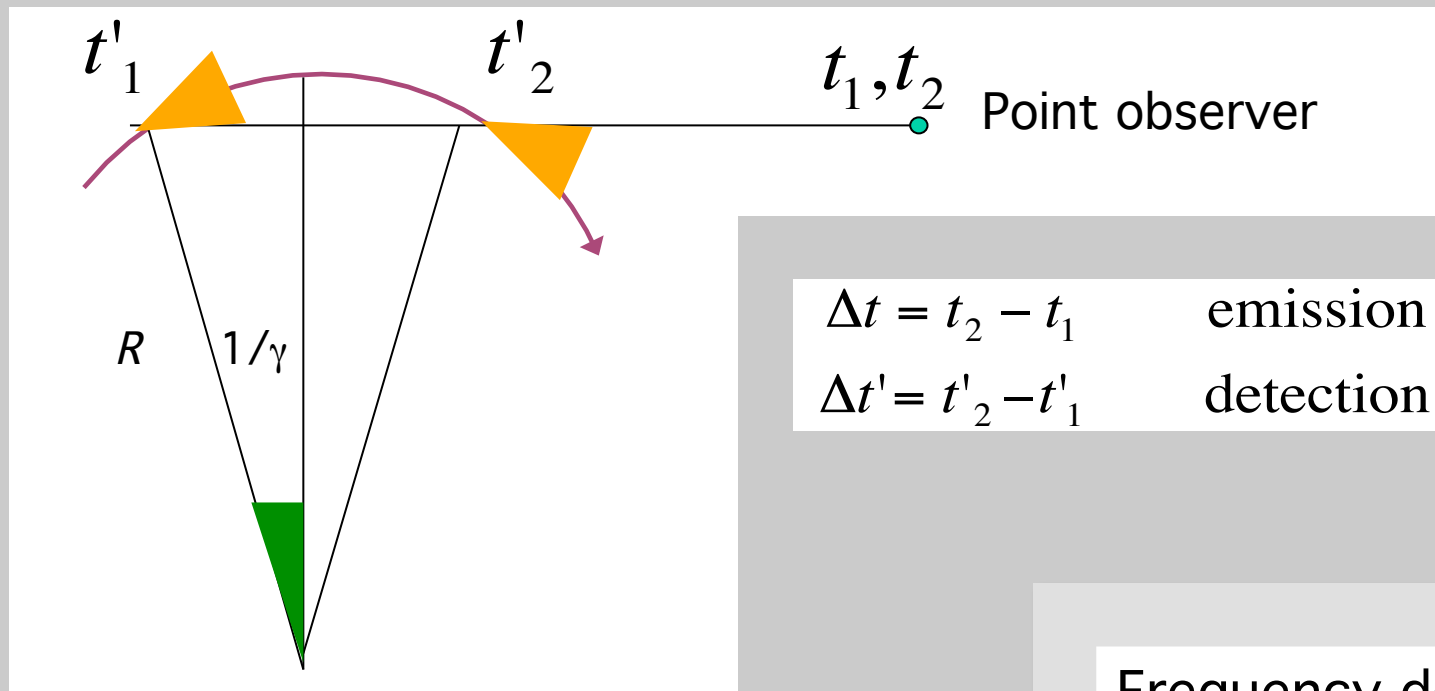


Frequency (energy) shift

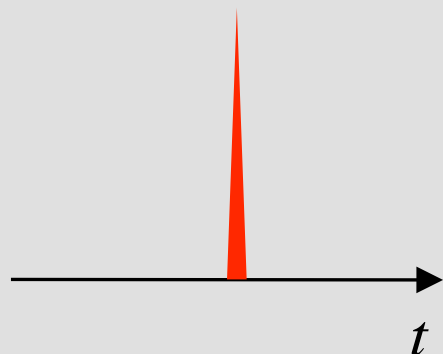
$$\omega_d = \omega_s \sqrt{\frac{1 + v/c}{1 - v/c}} = \omega_s \gamma (1 + v/c)$$



# S.R. spectral properties (a)



Time domain



Short time pulse

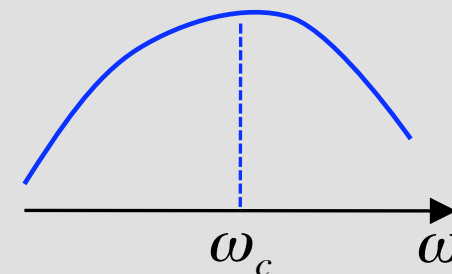
$$\Delta t \cong \frac{4}{3} \frac{R}{c\gamma^3}$$

Frequency domain

High frequencies

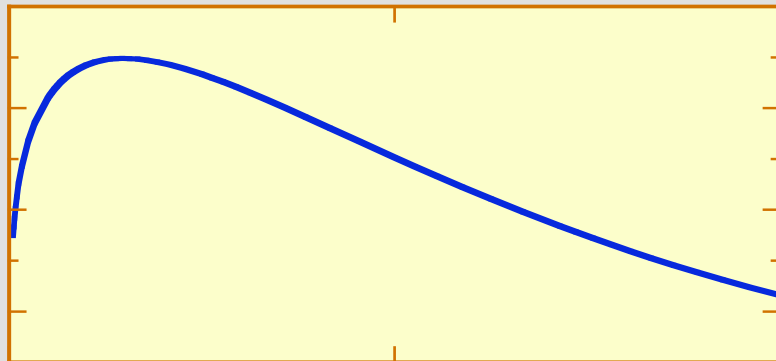
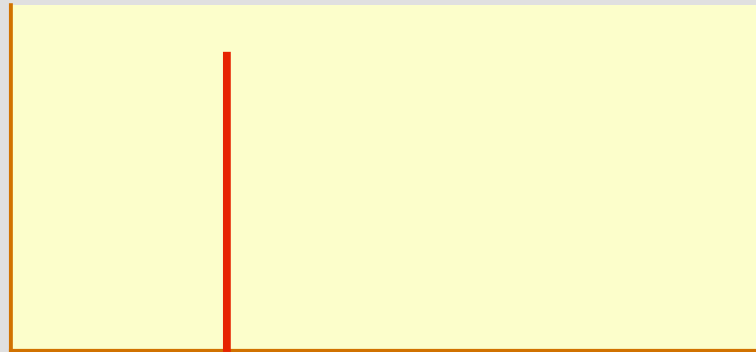
$$\omega_c \approx \frac{3c\gamma^3}{2R}$$

Broad spectrum



# S.R. spectral properties (b)

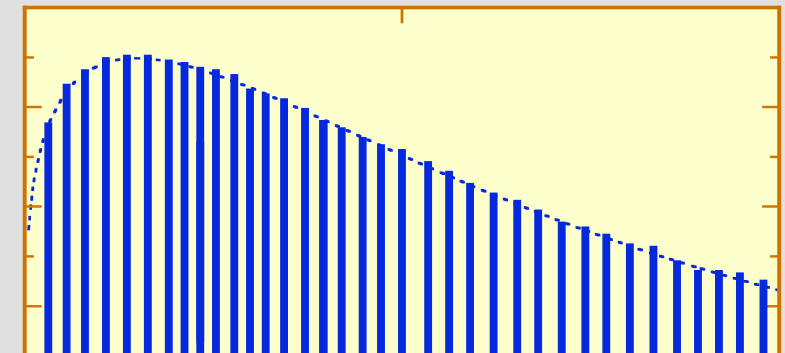
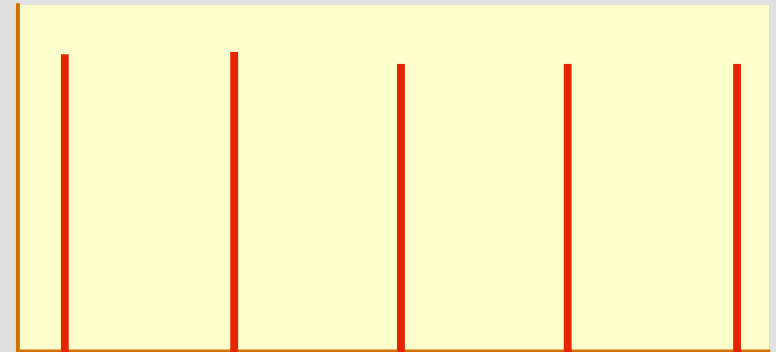
Single emission  
from circular arc



$\omega_c$

Time

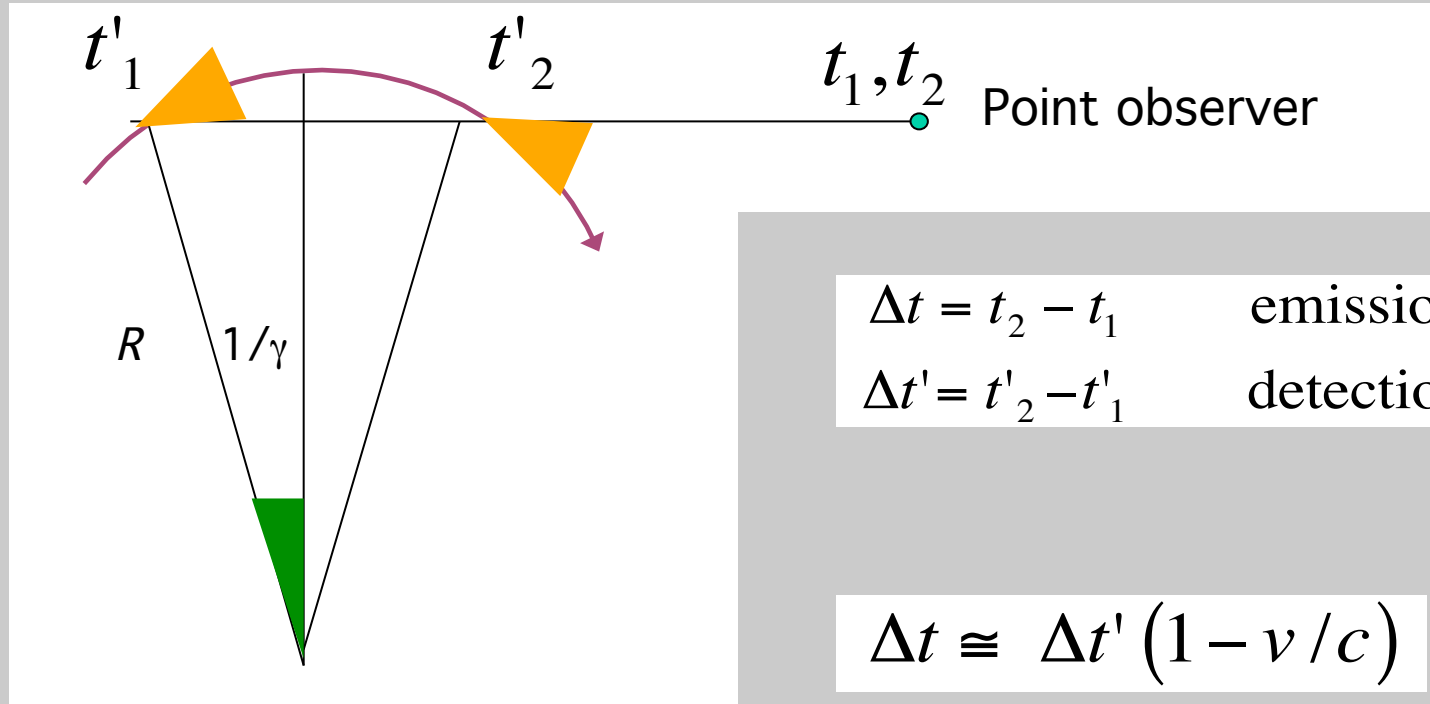
Periodic emission  
from a circular  
trajectory



$\omega_{rev}$

$\omega_c$

Frequency



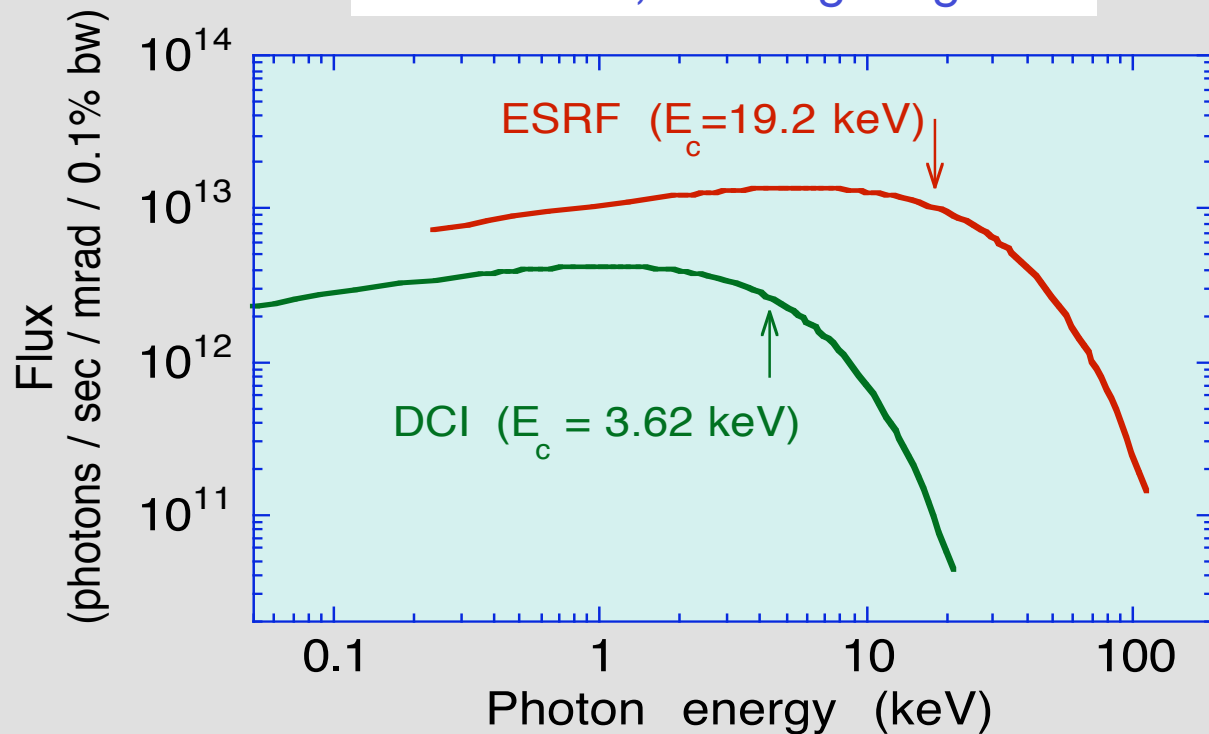
Time compression  $\Rightarrow$  Enhancement of intensity

# S.R. emission spectra

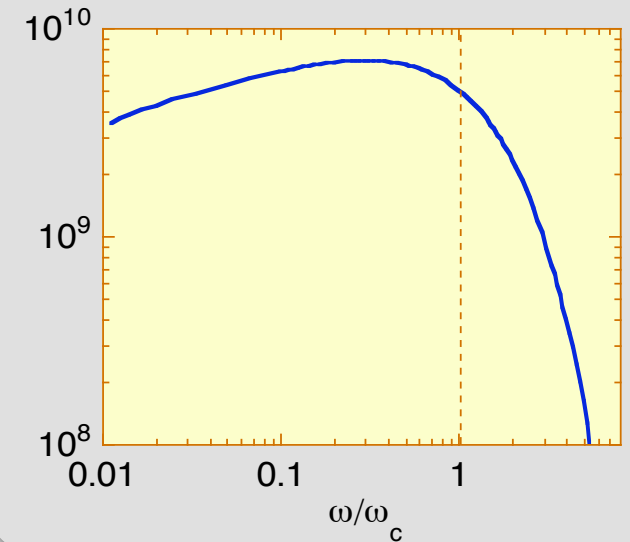
Photons/s/mrad/ 0.1%  $\Delta\lambda/\lambda$

$$\text{Flux} = I \gamma F_1$$

$I=100$  mA, bending magnets



$F_1 =$  S.R. universal curve



# S.R. polarization

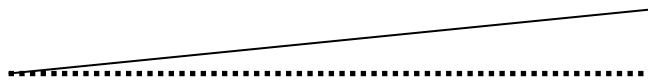
$\sigma$  - Horizontal component (in the orbit plane)

$\pi$  - The vertical component:  
increases with angle  
decreases with photon energy

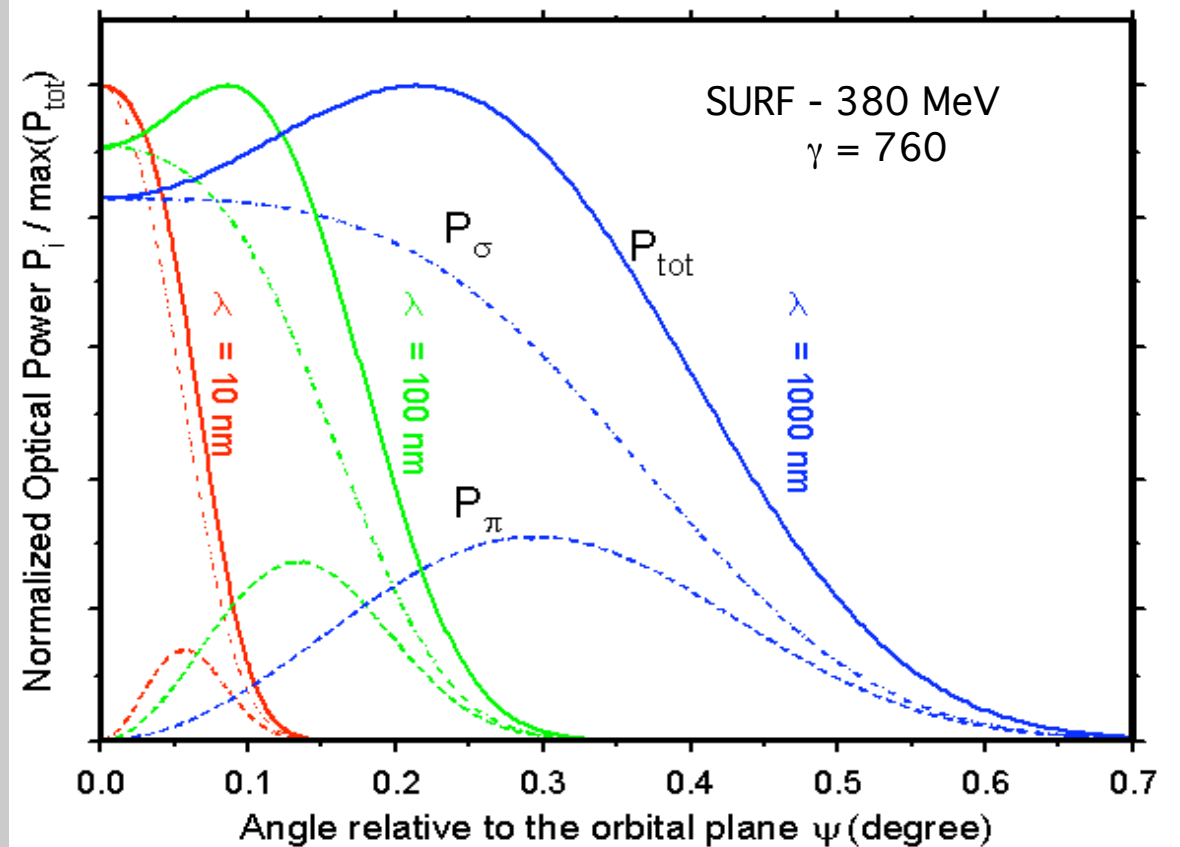
$\pm \frac{\pi}{2}$  dephasing

↓  
Elliptical polarisation

Vertical divergence



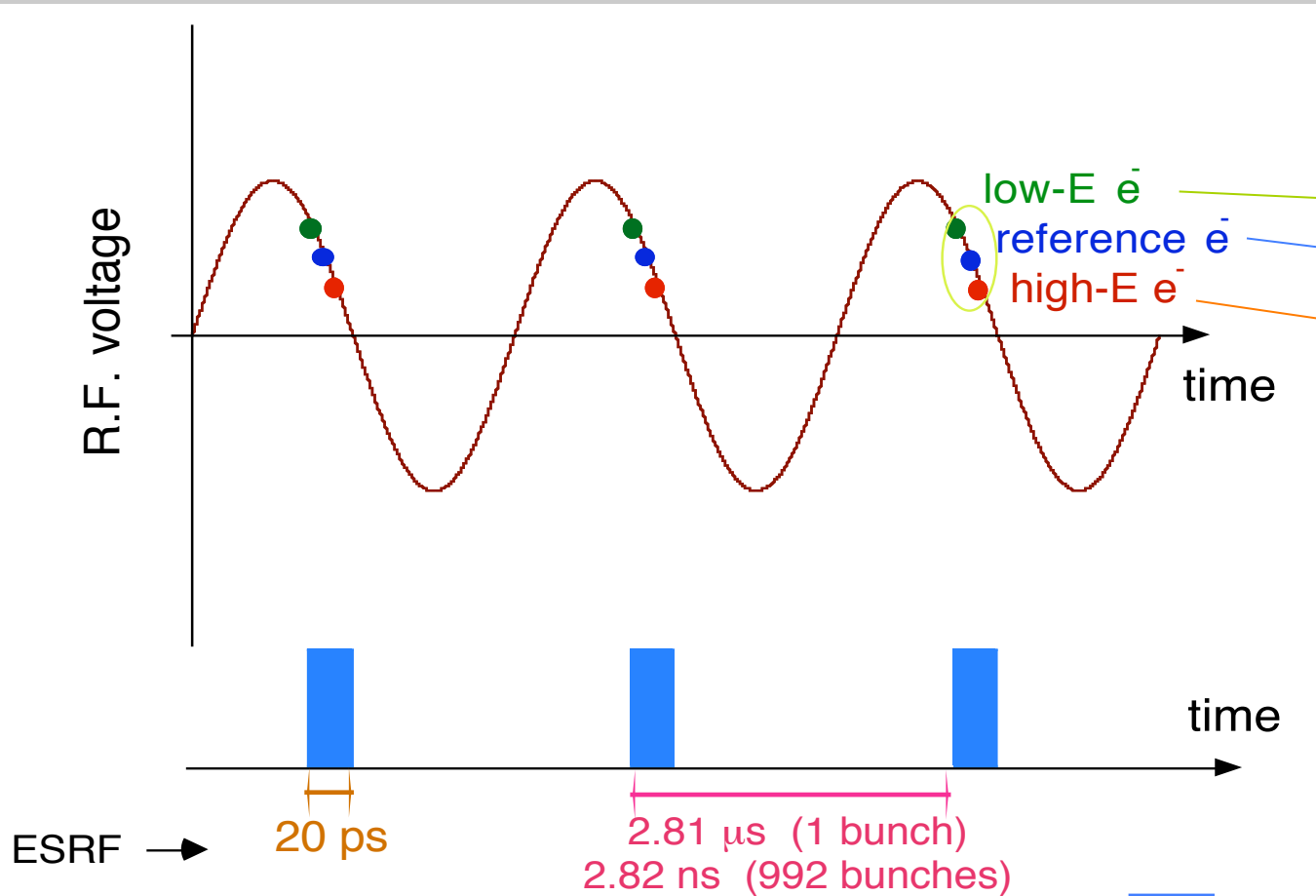
$$\psi \approx \frac{1}{\gamma} \quad \text{for} \quad E = E_c$$



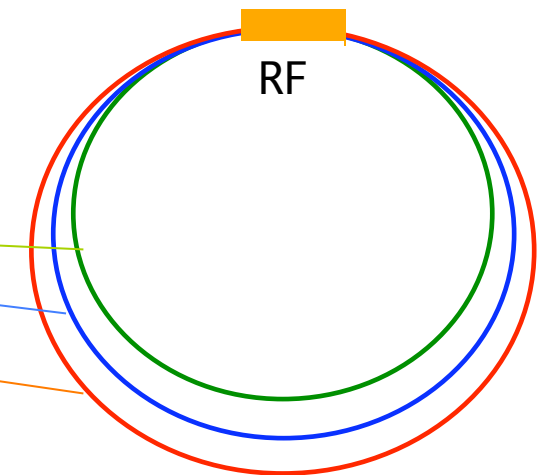


# S.R. time structure

## Phase-focussing in RF cavities



## Orbits



$$v \approx c; \quad \rho = \frac{m_0 c}{eB} \gamma$$

## Bunched structure:

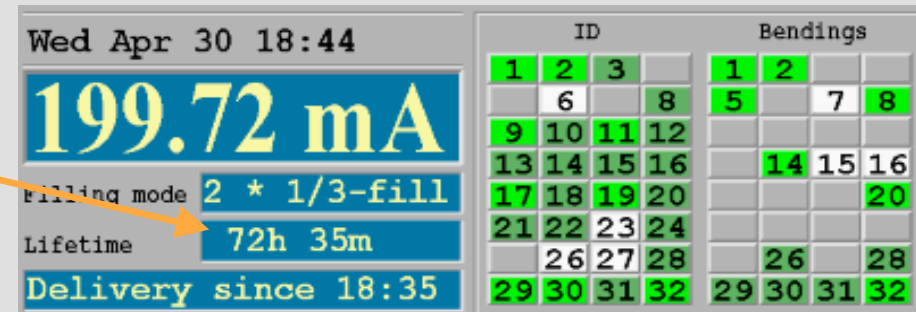
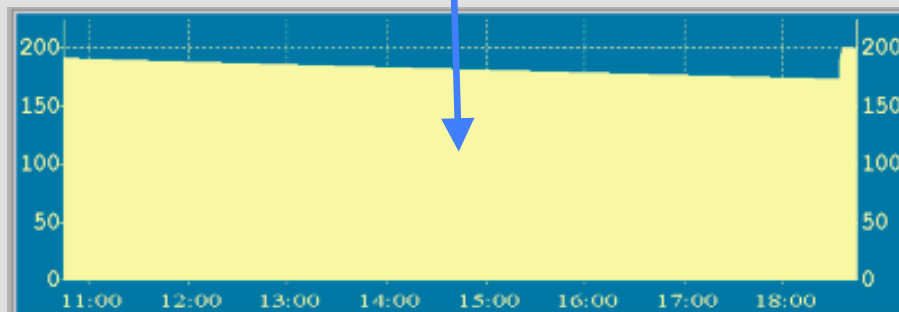
- of electron beam
- of S.R. emission

# $e^\pm$ beam lifetime

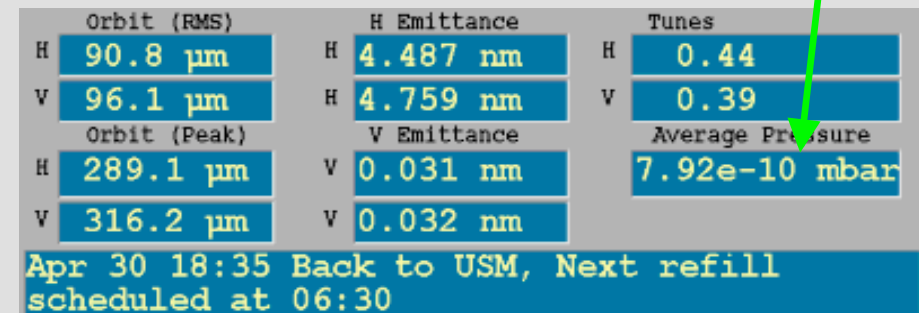
- Collisions with residual gas (photon-stimulated desorption)
- Occasional large energy losses through S.R. emission
- Non-linear resonances (anharmonic betatron oscillations)
- Toushek effect (e-e scattering inside each bunch)

$$\tau = \frac{I(t)}{dI/dt}$$

$$I(t) = I_0 \exp[-t/\tau]$$

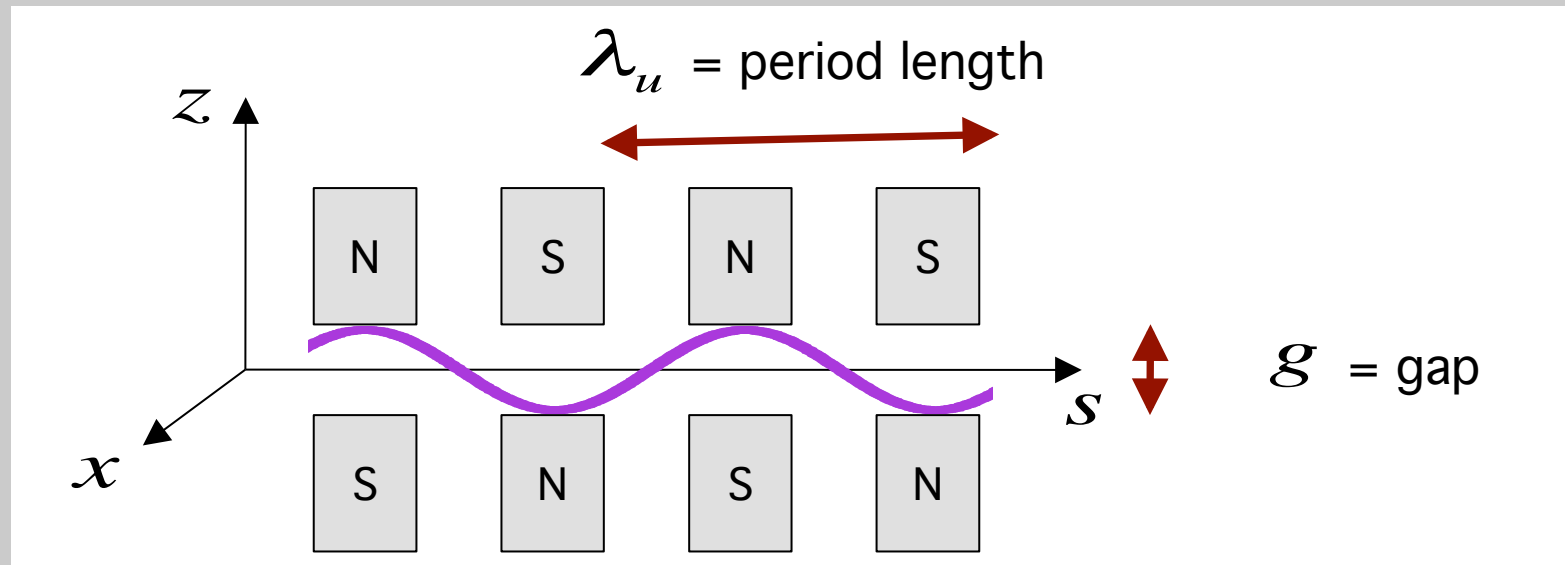


ESRF



**S. R. from insertion devices**

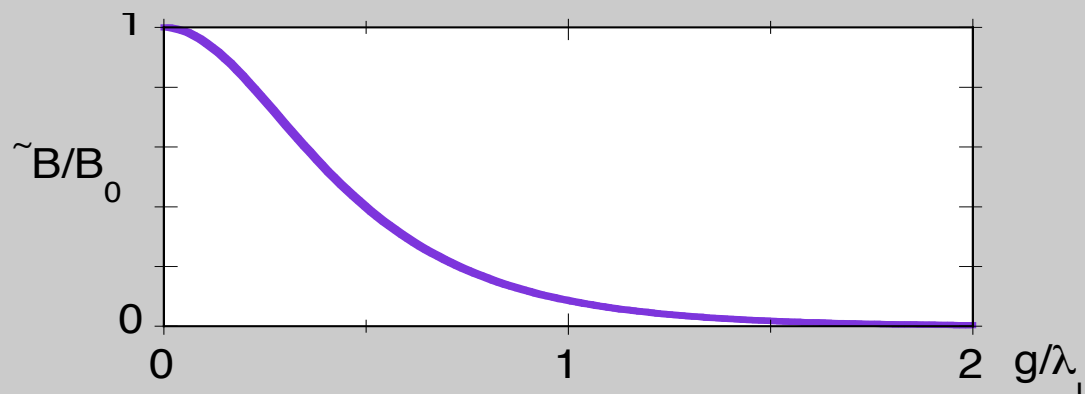
# Alternating magnetic fields



Vertical magnetic field

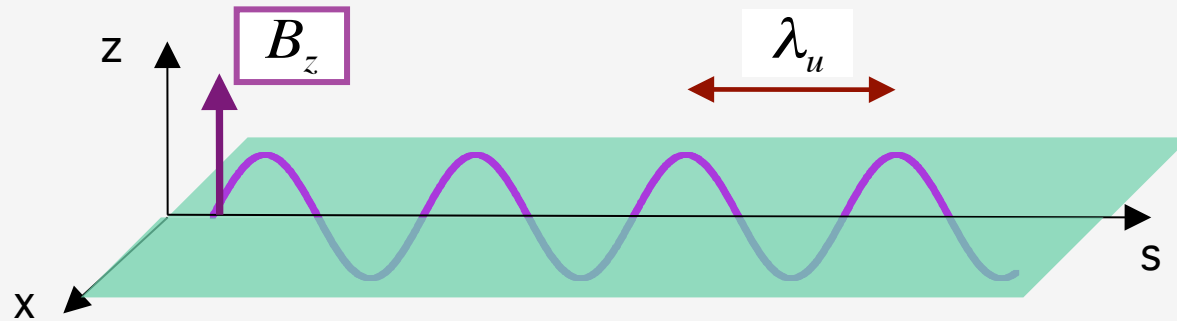
$$B_z(s) = \frac{B_0}{\cosh(\pi g / \lambda_u)} \cos(k_u s) = \tilde{B} \cos(k_u s)$$

Gap-period  
relation



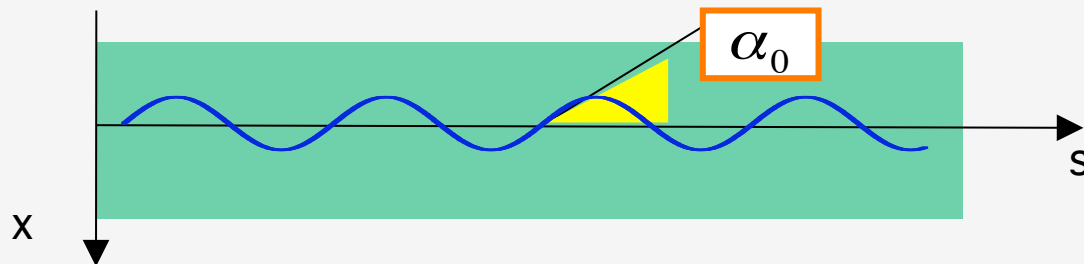
$$k_u = \frac{2\pi}{\lambda_u}$$

# Magnetic field effects



Oscillating  
vertical magnetic field

$$\vec{F} = e\vec{v} \times \vec{B}$$



Transverse beam oscillation

$$\alpha_0 = K \frac{1}{\gamma}$$

Wiggler/undulator  
parameter

$$K = \frac{eB\lambda_u}{2\pi c m_e}$$

# The K parameter

W/U properties

SR divergence

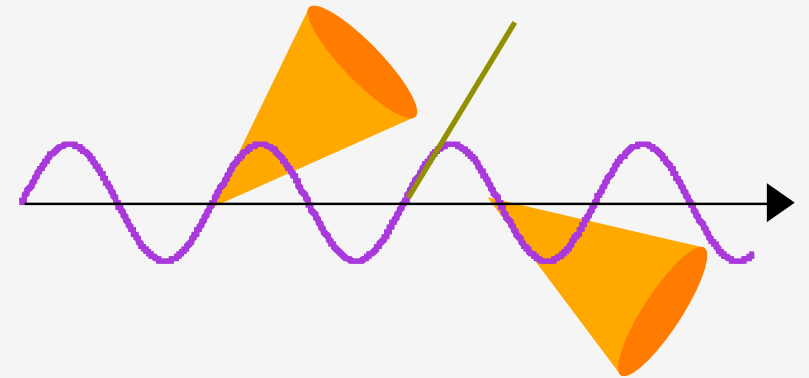
$$\alpha_0 = K \frac{1}{\gamma}$$

$$K = \frac{eB\lambda_u}{2\pi cm_e} = 0.934 \times B[\text{T}] \times \lambda_u[\text{cm}]$$

$$K > 1, \quad \alpha_0 > \frac{1}{\gamma}$$

High-K devices

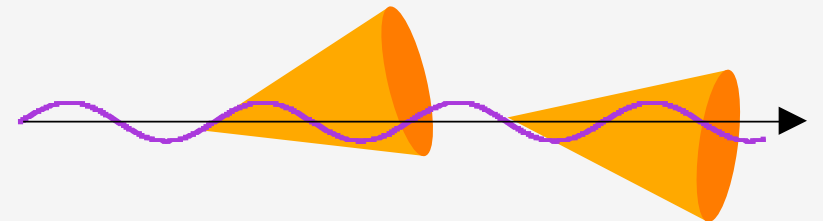
WIGGLERS



$$K \leq 1, \quad \alpha_0 \leq \frac{1}{\gamma}$$

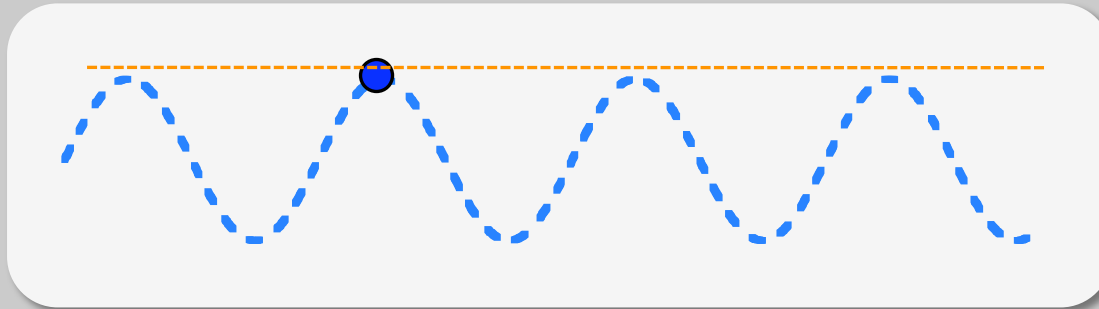
Low-K devices

UNDULATORS

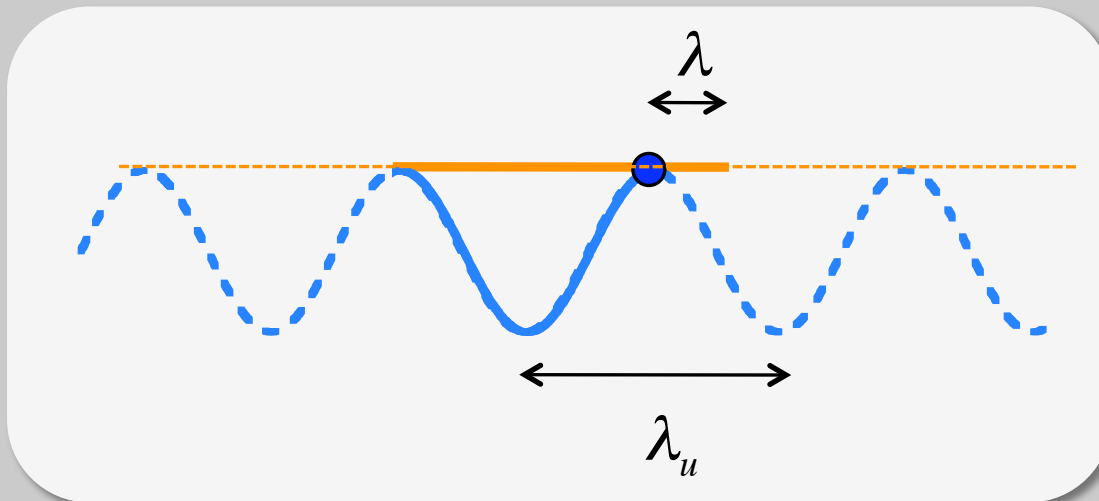


# Undulator: interference effect

time  $t_1$



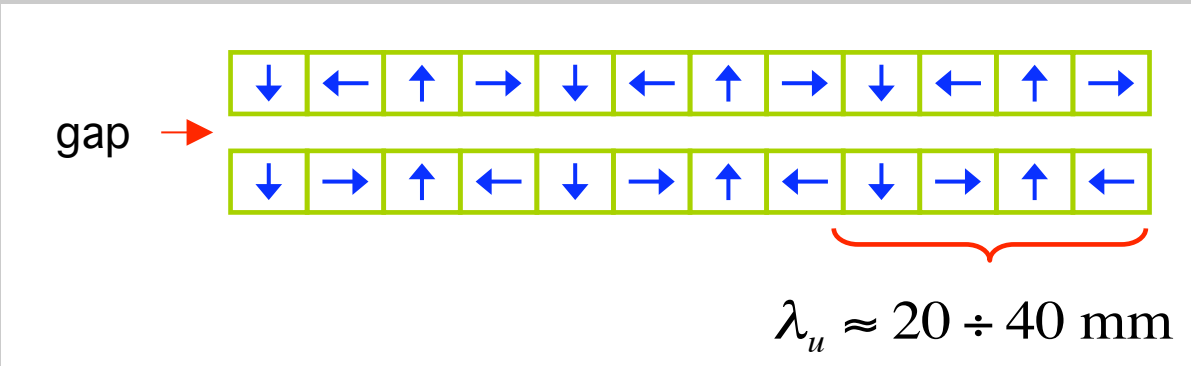
time  $t_2$



Constructive interference for

$$\lambda \approx \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

# Properties of undulator radiation (a)



$$\lambda \approx \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

tunability:  
gap  $\Rightarrow$  B  $\Rightarrow$  K

Angular  
red-shift

- Large N
- $K < 1$



Interference  
(monochromatic radiation)



# Properties of undulator radiation (b)

- Interference → Flux  $\propto N^2$  (in forward direction)

$N$  = number of wiggles

- Electron motion not perfectly sinusoidal → higher order harmonics

$$\omega_1 \approx \frac{4\pi c \gamma^2 / \lambda_u}{1 + K^2 / 2 + \gamma^2 \theta^2}$$

$$\omega_n = n\omega_1$$

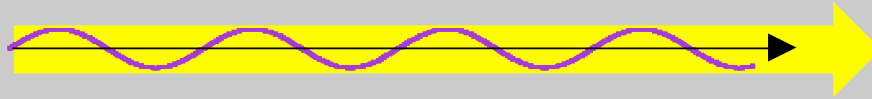
- Finite wave-train of radiation emitted by each electron → bandwidth

$$\frac{\Delta\omega_n}{\omega_n} \approx \frac{\gamma^2 \delta(\theta^2)}{1 + K^2 / 2}$$

$$\frac{\Delta\omega_n}{\omega_n} \approx \frac{1}{nN}$$

# Low-K .vs. high-K devices

Low-K



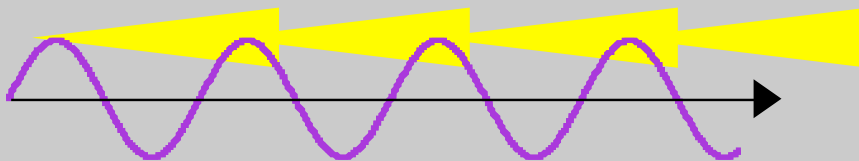
Coherent superposition  
Interference

$$\lambda \approx \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

Increasing K:

- $\lambda$  increases
- relevance of harmonics
- interference effects reduced
- broader spectrum

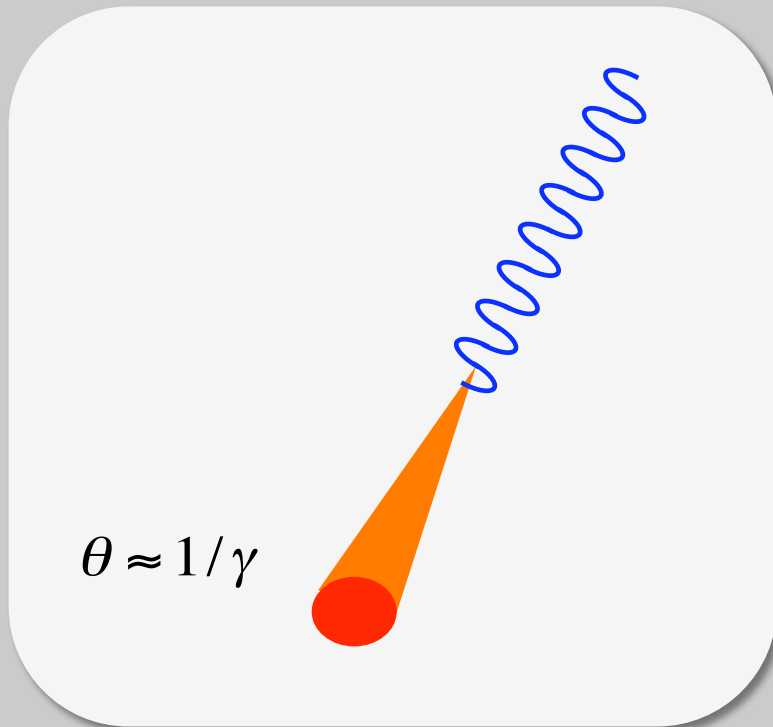
High-K



Incoherent superposition  
Continuous spectrum

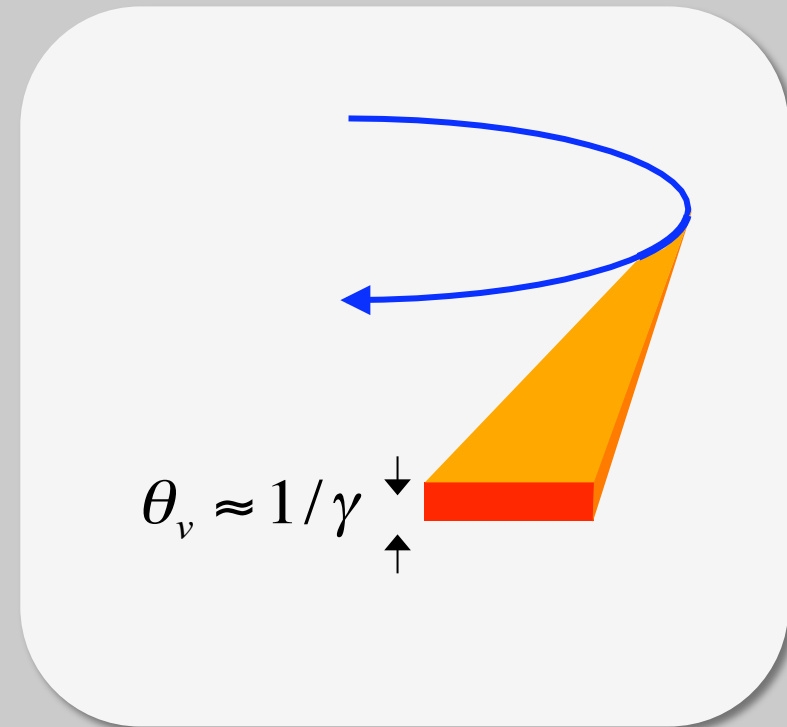
# Undulators .vs. bending magnets (a)

Undulator



Vertical and horizontal collimation

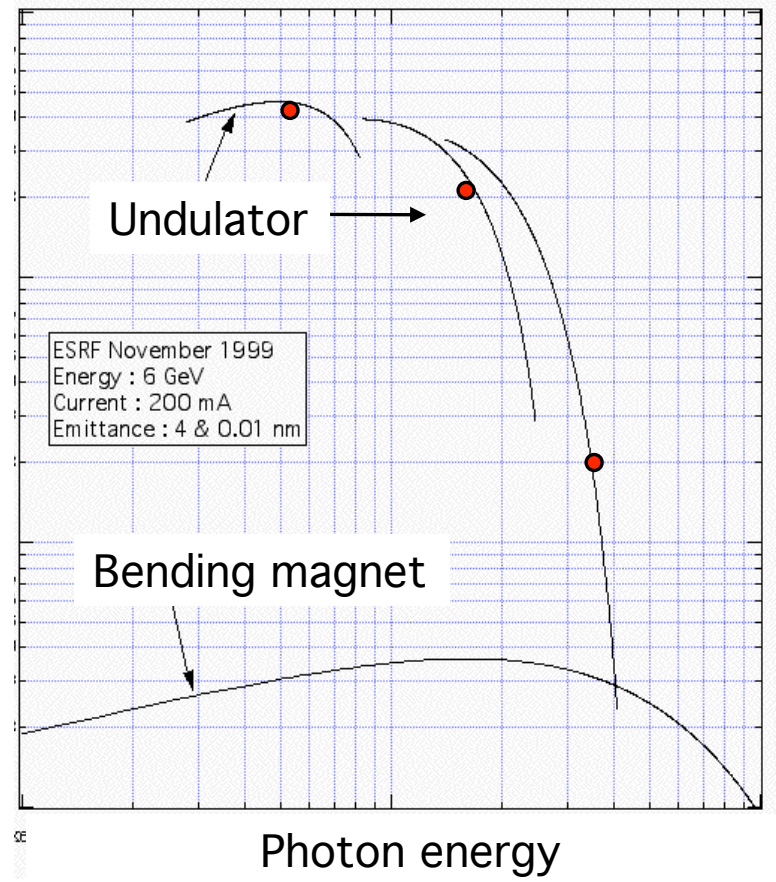
Bending magnet, wiggler



Vertical collimation

# Undulators .vs. bending magnets (b)

Flux



## Undulator

Strong emission at discrete energies (fundamental + harmonics)

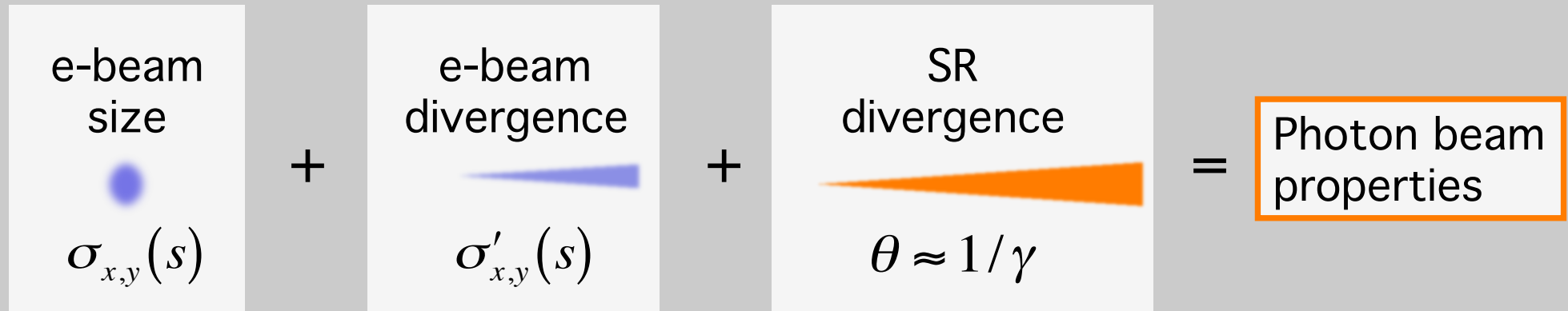
Tunability by varying the gap

## Bending magnet, wiggler

Emission over a continuous spectrum

# Brilliance of S.R.sources

# Photon beam properties



ESRF	Horizontal		Vertical	
	$\sigma_x$ [ $\mu\text{m}$ ]	$\sigma'_x$ [ $\mu\text{rad}$ ]	$\sigma_y$ [ $\mu\text{m}$ ]	$\sigma'_y$ [ $\mu\text{rad}$ ]
Even I.D.	415	51	8.6	2.9
Odd I.D.	10.3	108	8.6	2.9

S.R. divergence  $\theta \approx 1/\gamma \approx 100 \mu\text{rad}$

# Photon beam parameters

Brilliance

$$\frac{\text{photons}}{\text{s mm}^2 \text{ mrad}^2 \text{ bandwidth}}$$

↑                    ↑                    ↑

source size    solid angle    0.1%  $\Delta\lambda / \lambda$

Integrating over source size

+

Integrating over vert. angle

+

Integrating over horiz. angle

+

Integrating over bandwidth

Brightness

$$\frac{\text{photons}}{\text{s} \cdot \text{mrad}^2 \cdot \text{bandwidth}}$$

Spectral flux

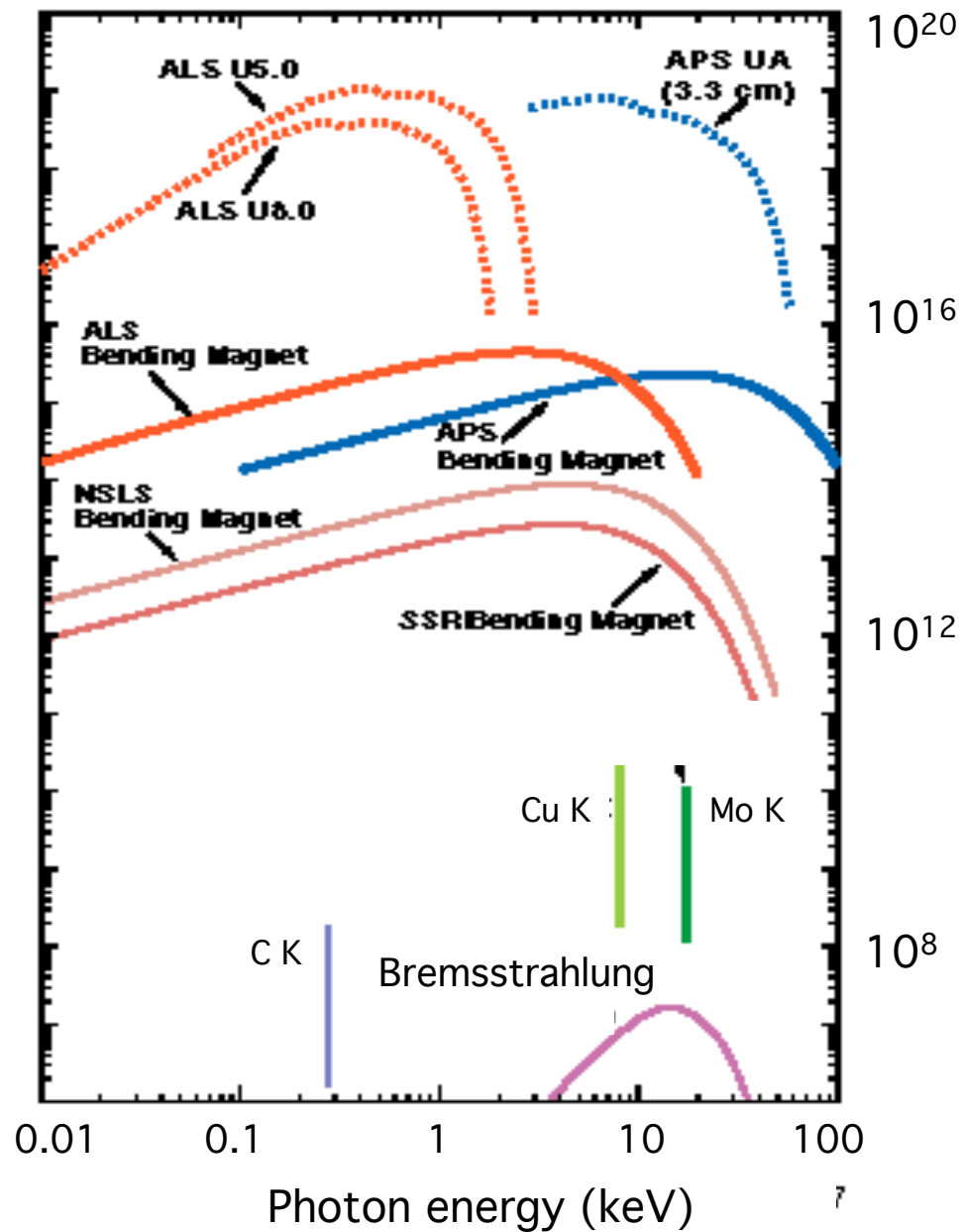
$$\frac{\text{photons}}{\text{s} \cdot \text{mrad} \cdot \text{bandwidth}}$$

Total flux

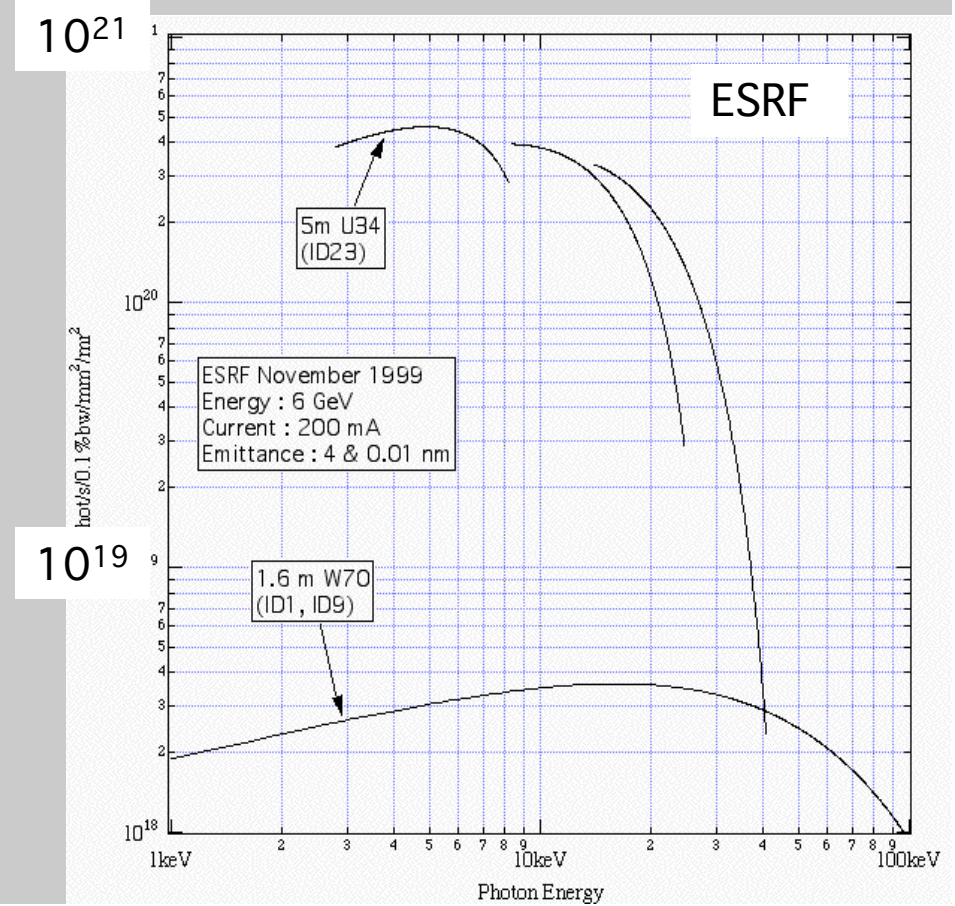
$$\frac{\text{photons}}{\text{s} \cdot \text{bandwidth}}$$

$$\frac{\text{photons}}{\text{s}}$$

# Brilliance: comparisons



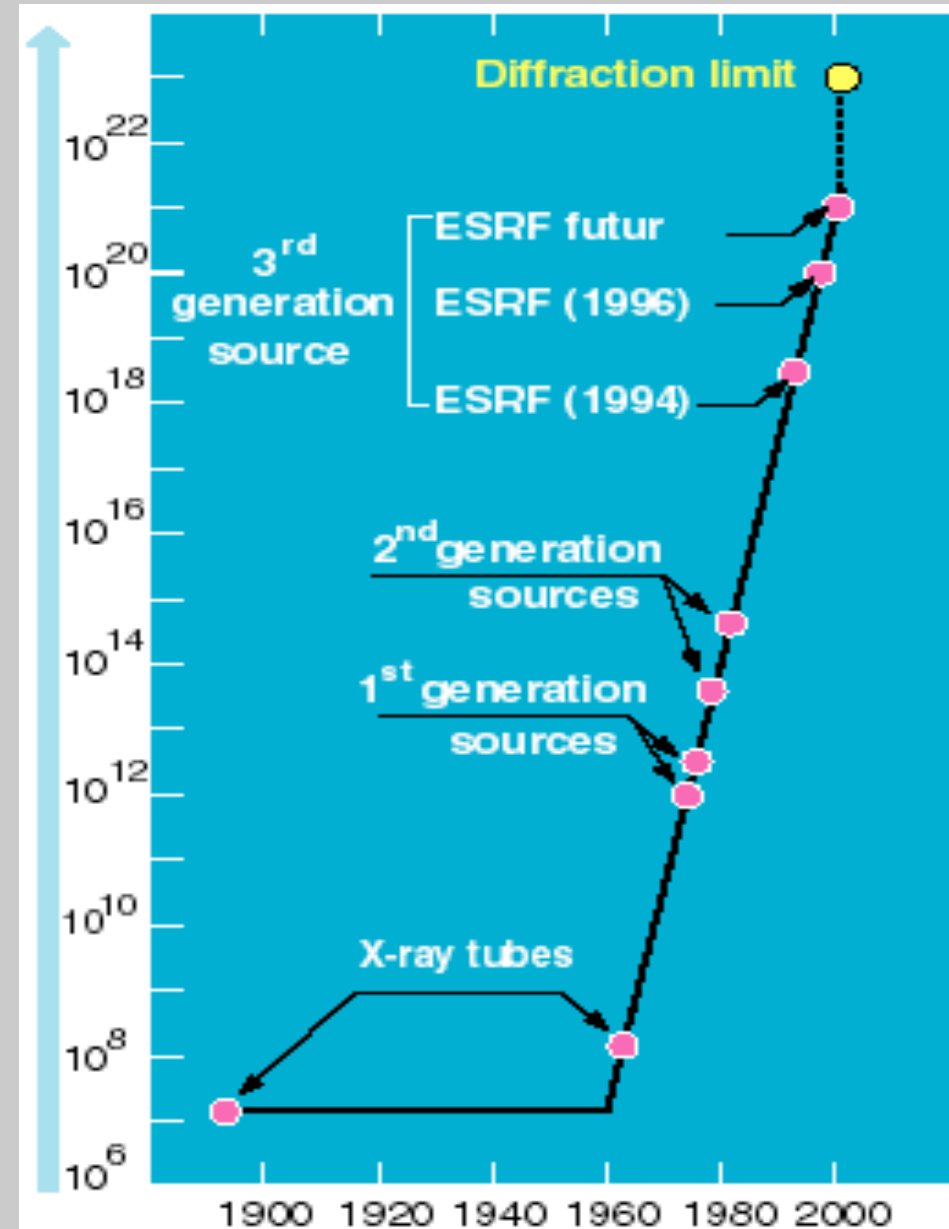
photons  
s mm<sup>2</sup> mrad<sup>2</sup> bandwidth





# Brilliance: time evolution

$$\frac{\text{photons}}{\text{s mm}^2 \text{ mrad}^2 \text{ bandwidth}}$$



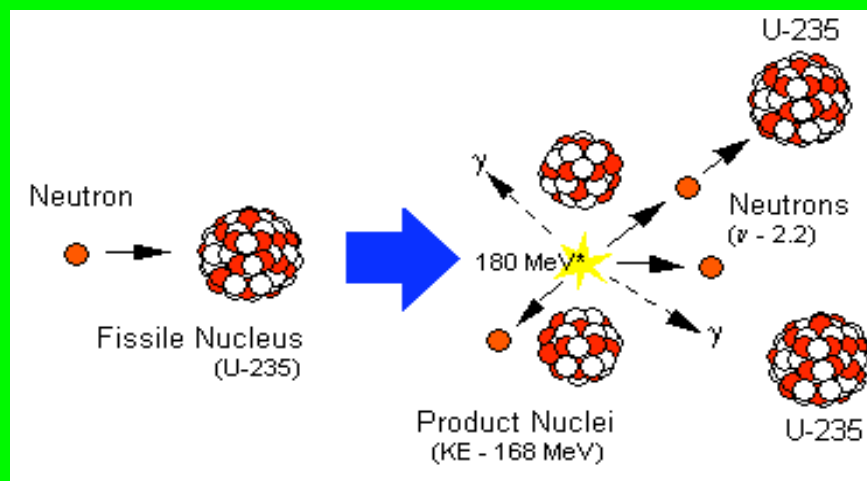
# Neutron sources

# Neutron production

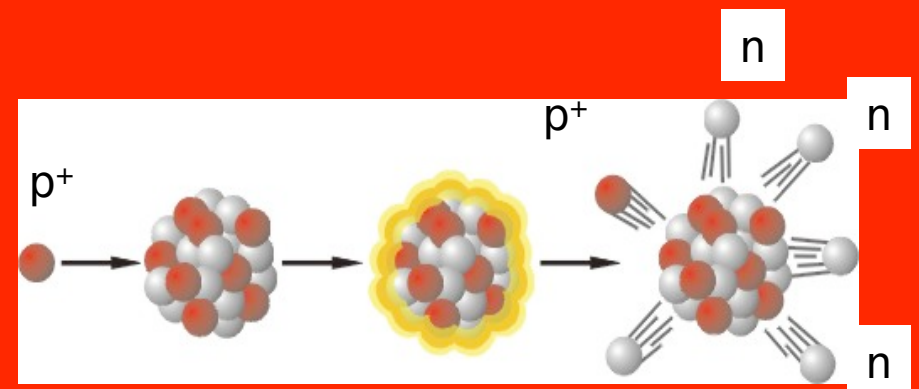
- Radioactive sources
- Neutrons from nuclear reactors
- Neutrons from pulsed accelerators
  - photofission
  - spallation

Most effective for solid state research:

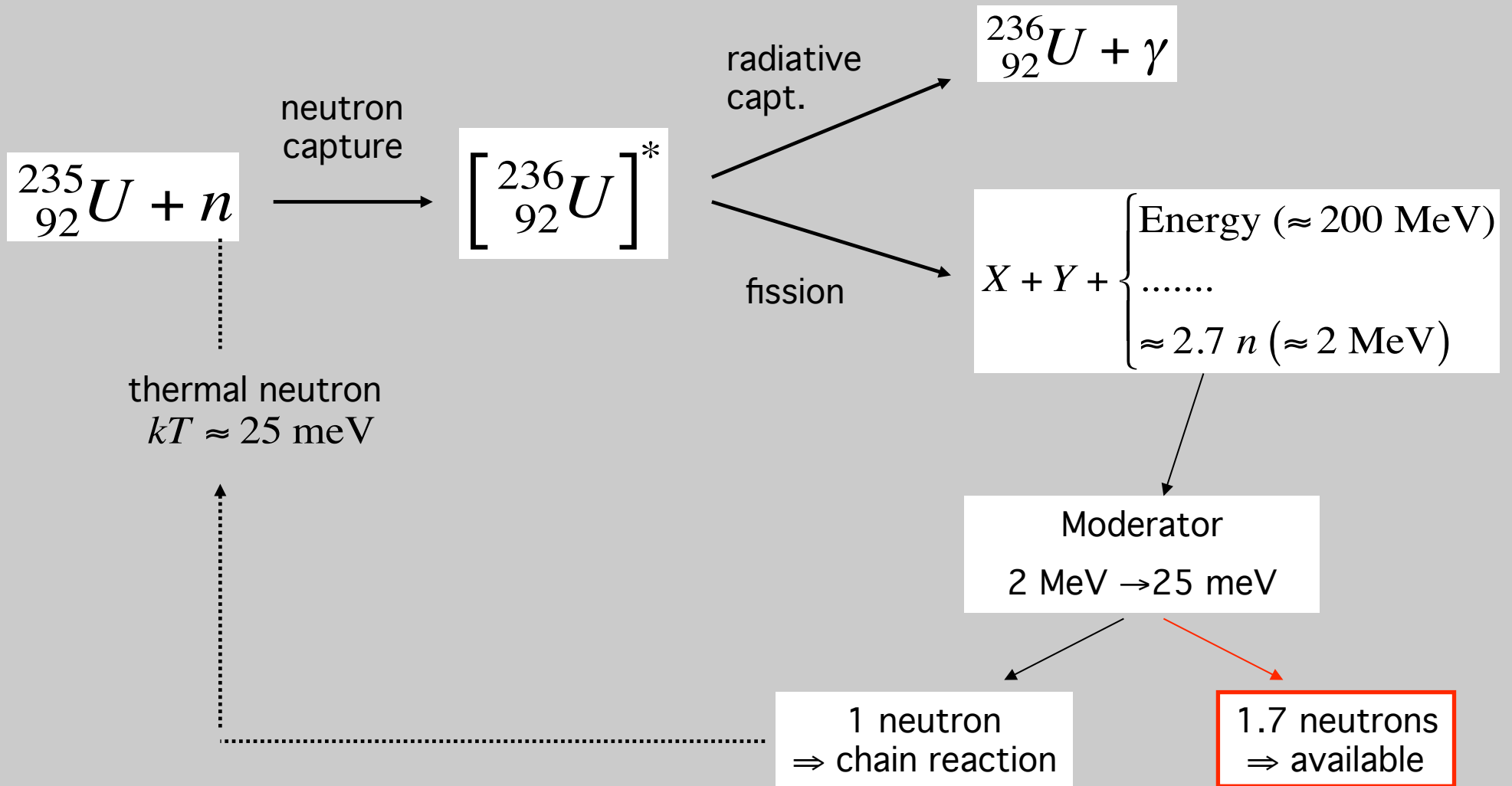
Fission (reactors)



Spallation (accelerators)



# Fission

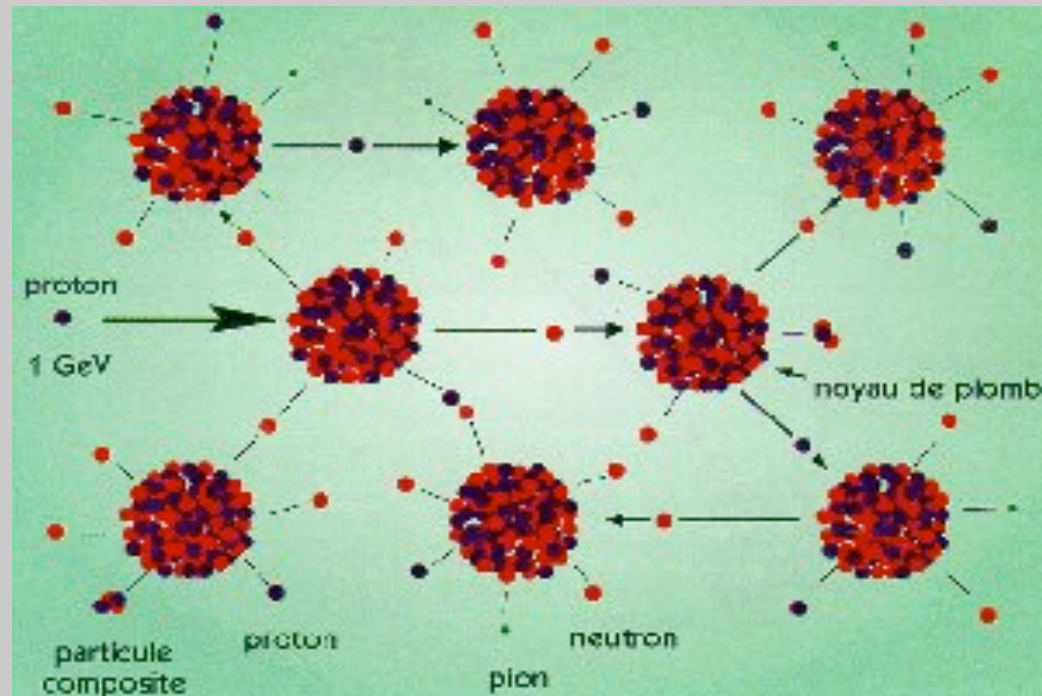


# Spallation

Proton accelerator

≈ 1 GeV proton

interaction  
with nucleons

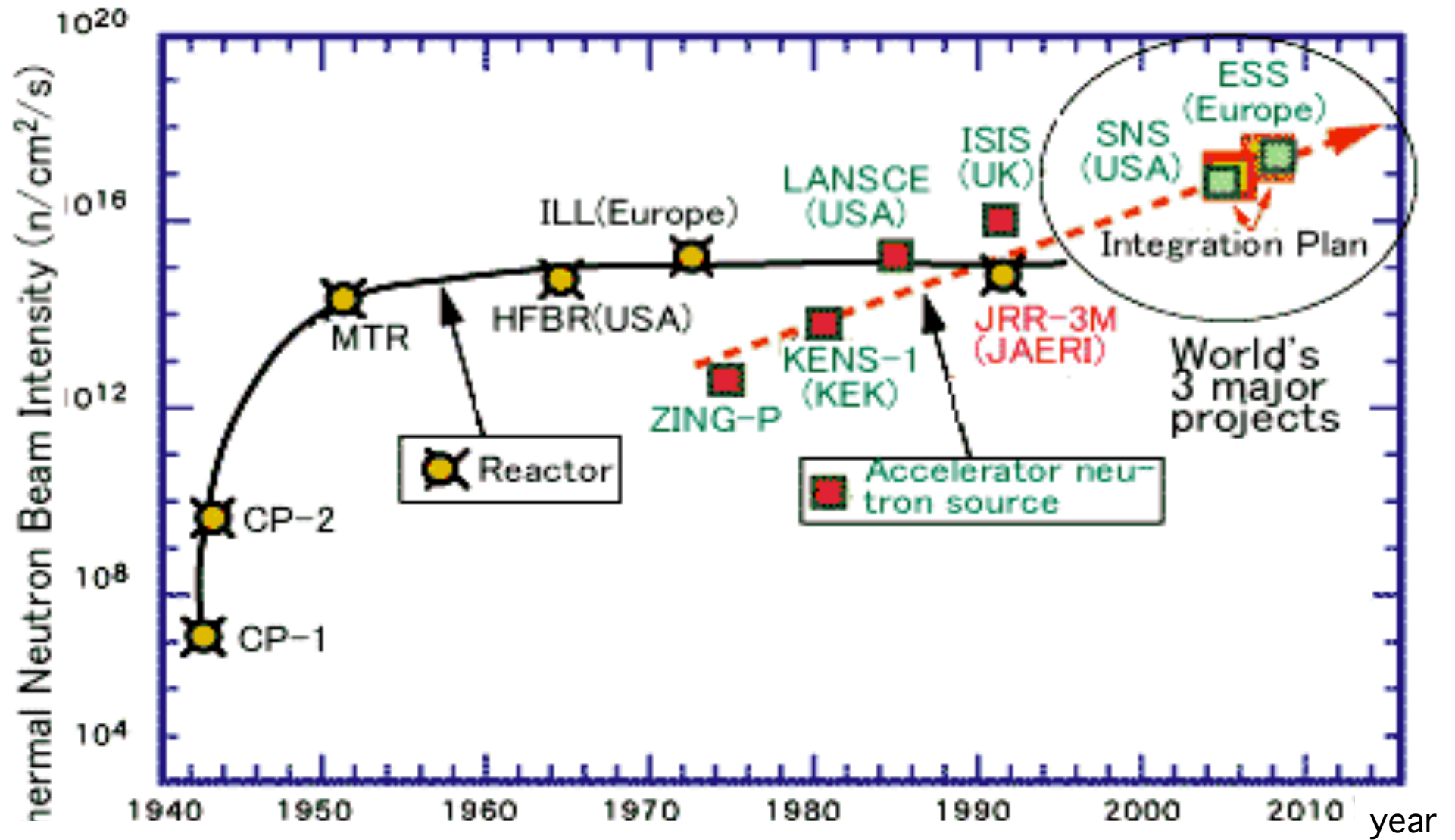


- high-energy particles
- chain-reactions
- excitations of nuclei
- de-excitation (evapor.)

20-40 neutrons  
(≈55 MeV)

- Concentrated source ⇒ high flux
- Pulsed source

# Neutron sources

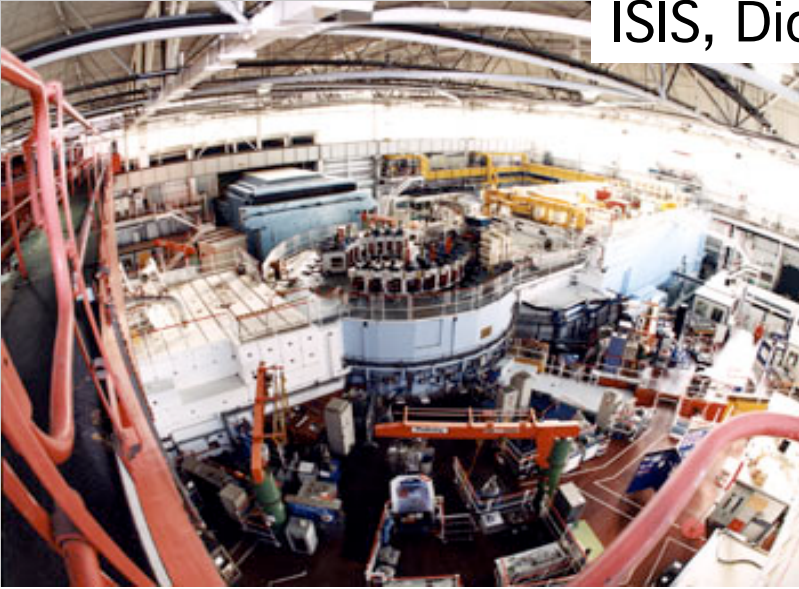
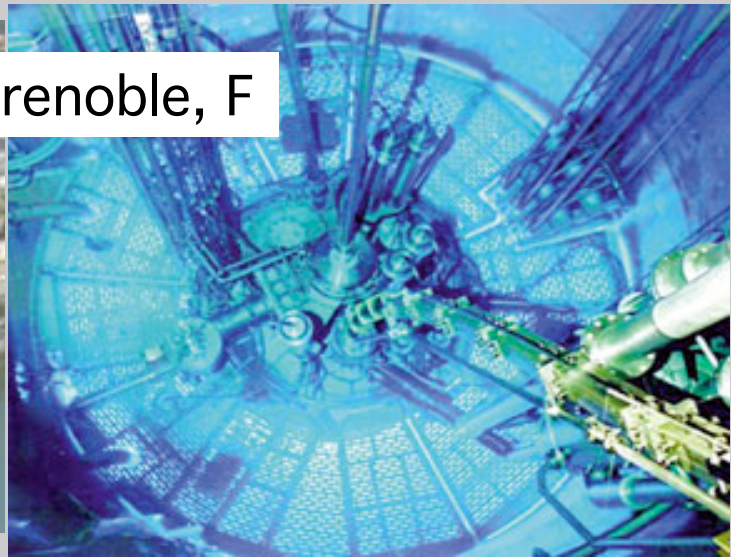


# Largest European Neutron Labs



ESRF(SR)

ILL Grenoble, F



ISIS, Didcot, UK

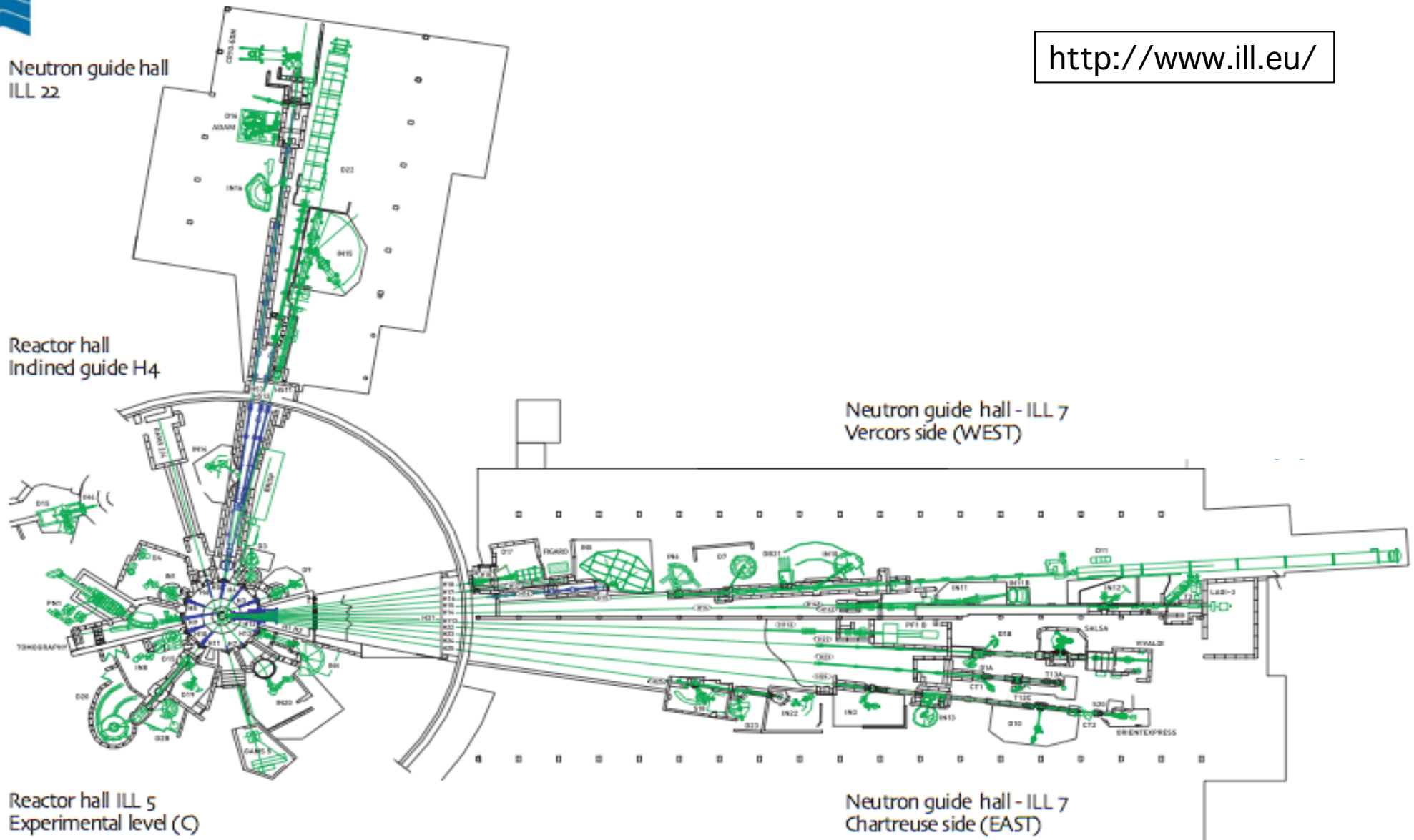


Diamond (SR)

# ILL reactor source

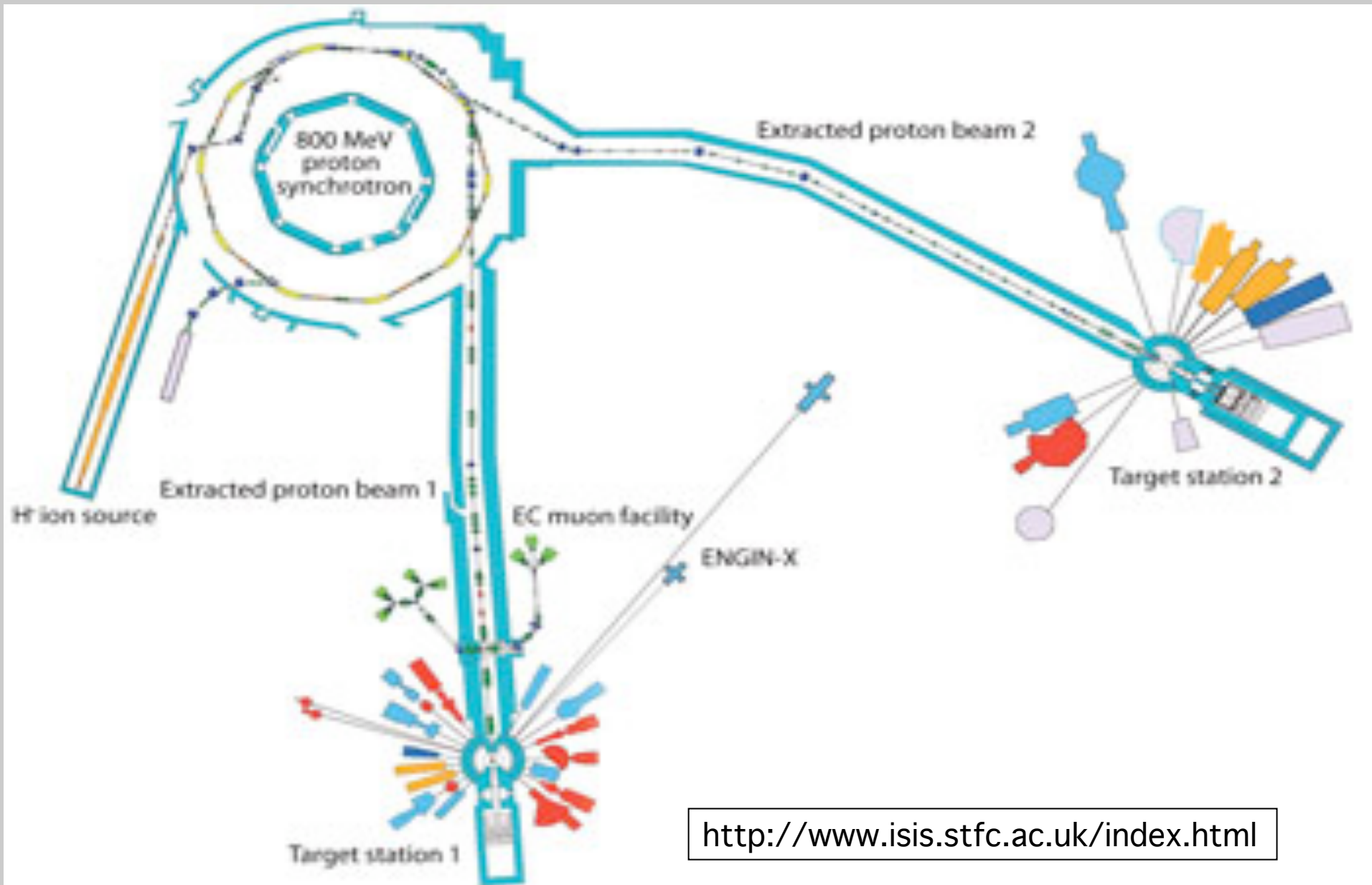


<http://www.ill.eu/>





# ISIS spallation source



<http://www.isis.stfc.ac.uk/index.html>

# Energy of neutrons

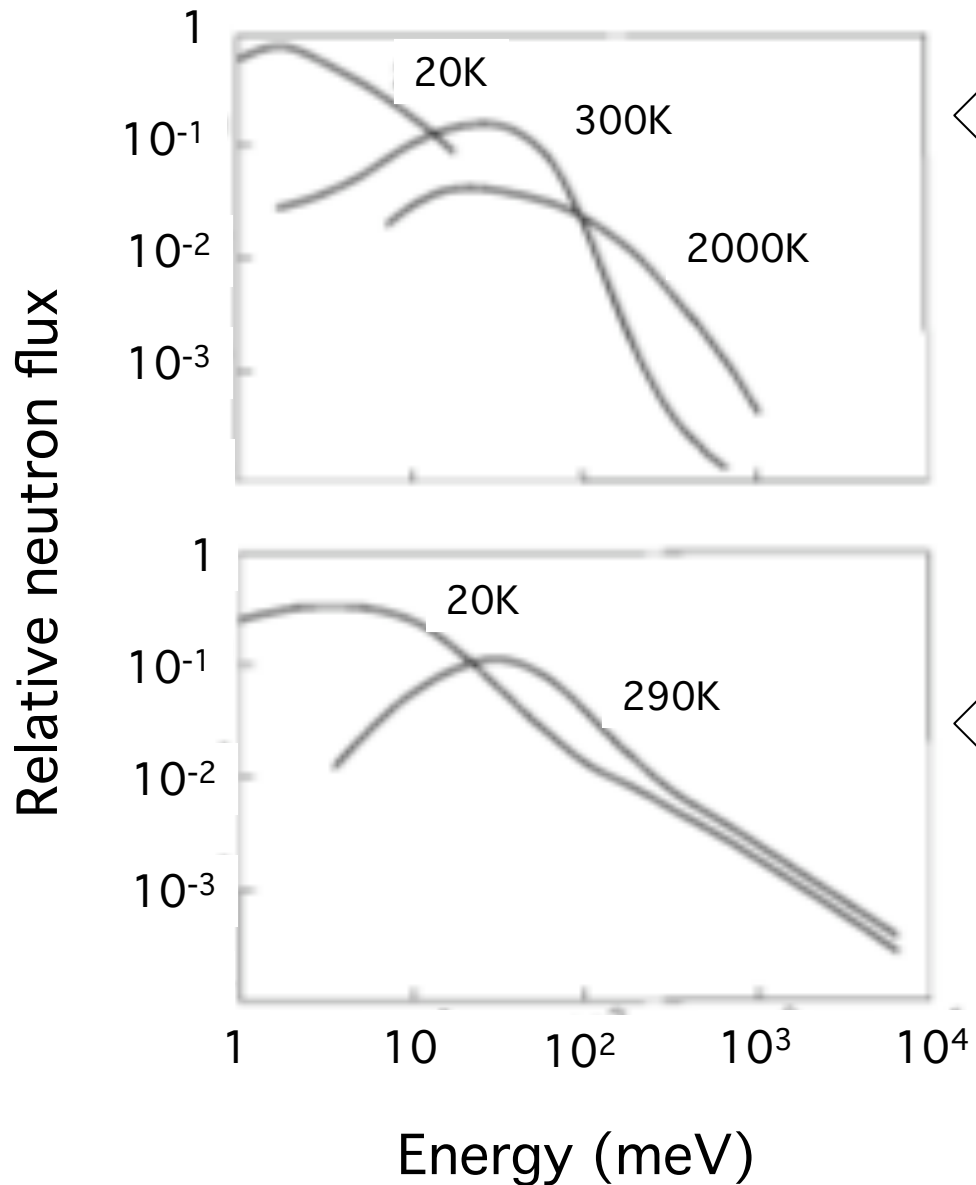
Moderation:

Neutrons are slowed down in moderators, where they are brought to thermal equilibrium through inelastic collisions with light atoms (H, D, Be)

The Table refers to the peak values of the Maxwell equilibrium distribution.

	<u>Ultra-cold</u>	<u>Cold</u>	<u>Thermal</u>	<u>Hot</u>	<u>Epi-thermal</u>
Energy (meV)	0.00025	1	25	250	1000
Temperature (K)	0.003	12	290	2900	12000
Wavelength (Å)	570	9	1.8	0.57	0.29
Velocity (m/s)	6.9	440	2200	6900	14000

# Energy distribution of neutrons



Reactor sources

CONTINUOUS PRODUCTION  
Energy selection  
by crystal monochromators  
(Bragg law)

Spallation sources

Higher energies available

PULSED PRODUCTION  
Energy selection  
by time-of-flight techniques

# Neutrons and X-rays properties (b)

	Thermal neutrons	X-rays (synchrotron)	X-rays (anodes)
Energy (eV)	$10^{-1}$ eV	$10^4$ eV	$10^4$ eV
Wavelength (Å)	0.5 - 10	0.5 - 5	0.5 - 2
Flux (part./cm <sup>2</sup> /s)	$10^{10}$ - $10^{15}$	$10^{20}$ - $10^{25}$	$10^{16}$ - $10^{17}$
Sample volume (mm <sup>3</sup> )	1 - 10	$10^{-5}$ - $10^{-1}$	$10^{-3}$ - $10^{-2}$
Tunability	yes	yes	no
Beam divergence	5 mr	$10^{-1}$ mr	5 mr
$\Delta\lambda/\lambda$	$10^{-3}$	$10^{-5}$ - $10^{-4}$	$10^{-3}$
Absorption	weak	medium	medium