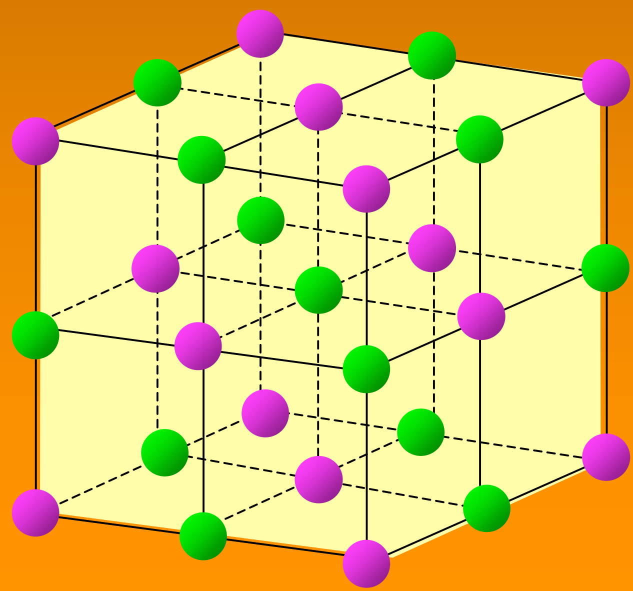
African School and Workshop
on X-Rays in Materials
some established techniques and practical applications

Dakar, Senegal
December 12-17, 2005

Basic crystallography



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Overview

- X-rays
- Crystals
- Crystal lattices
- Some relevant crystal structures
- Crystal planes
- Reciprocal lattice
- Crystalline and non-crystalline materials

X-rays

1895 - Discovery of X-rays

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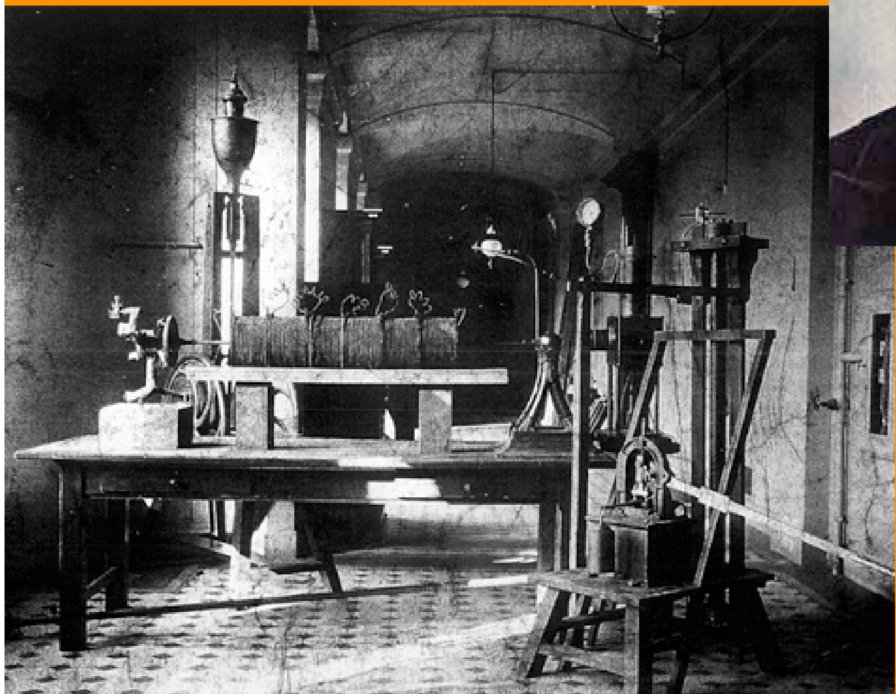
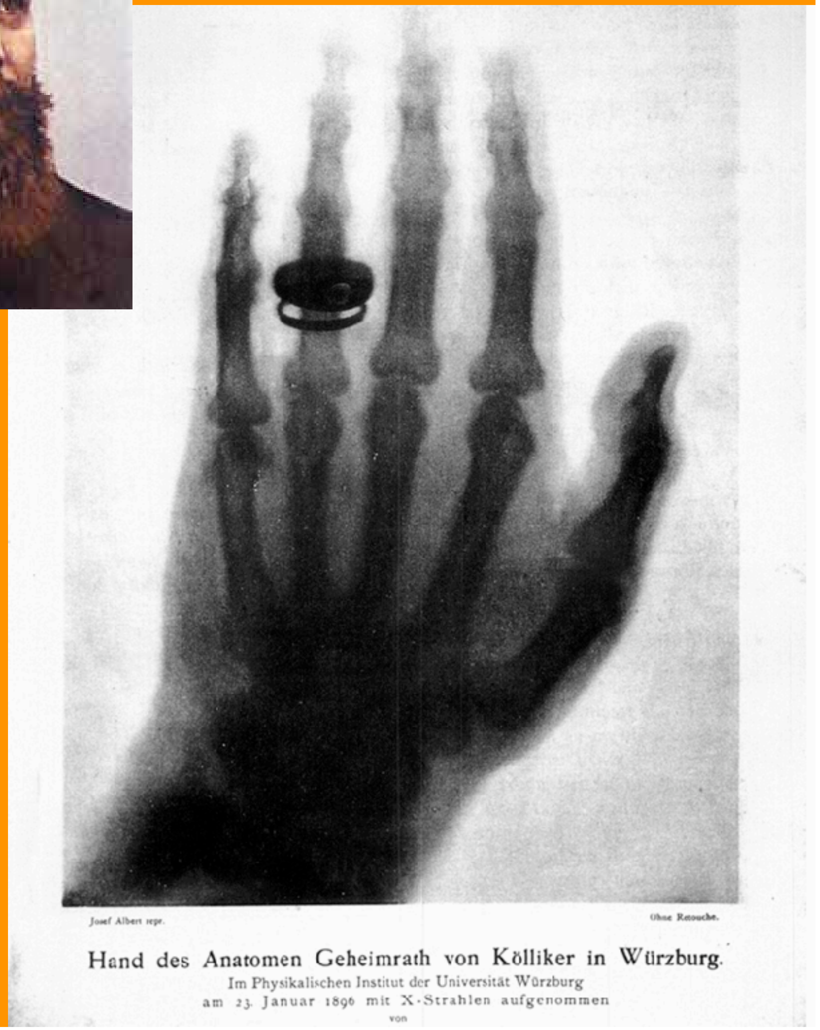
Wilhelm Konrad Röntgen
(1845-1923)



Würzburg (Germany)
November 8,1895



Electromagnetic waves

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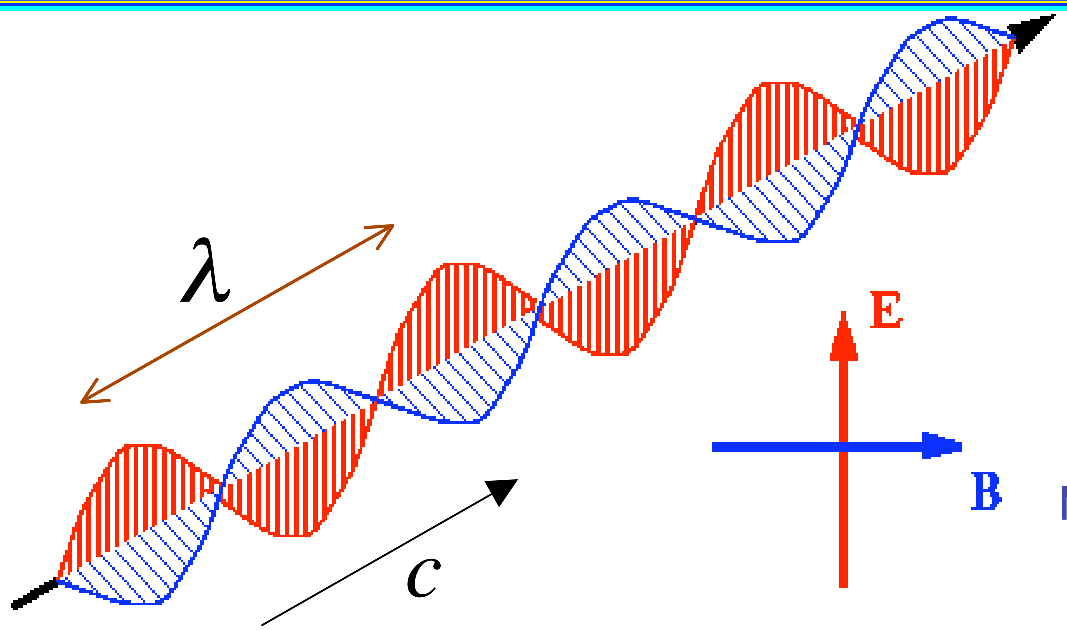

Electric field

Magnetic field

Speed (in vacuum):

$$c \approx 3 \times 10^8 \text{ m/s}$$

1 Å = 10^{-10} m
1 nm = 10^{-9} m

Wavelength	λ	
Frequency	$\nu = c/\lambda$	$\omega = 2\pi\nu$
Photon energy	$E = h\nu = \hbar\omega$	

Wave-vector

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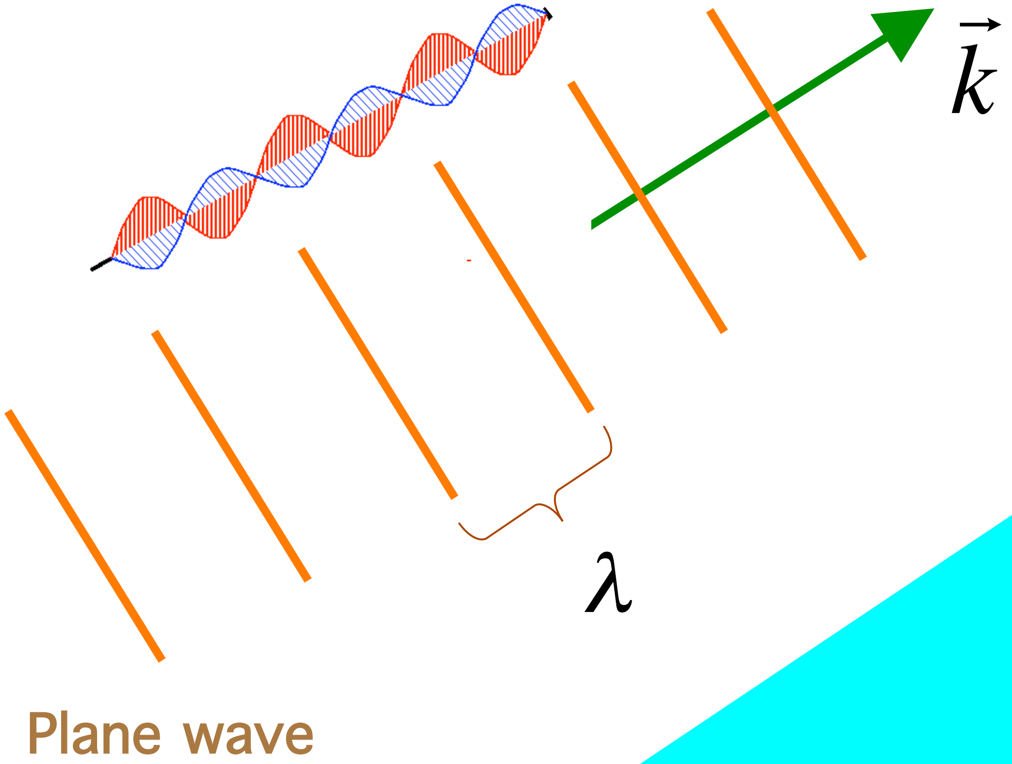
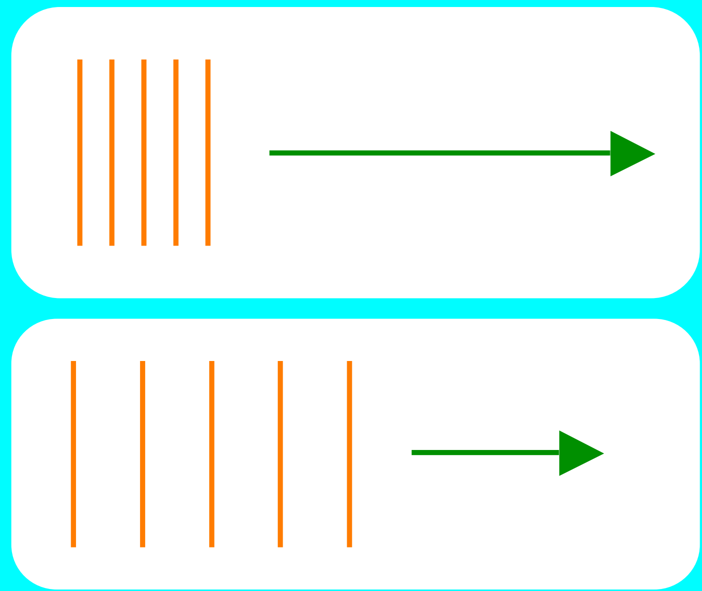


$$k = \frac{2\pi}{\lambda}$$



Plane wave

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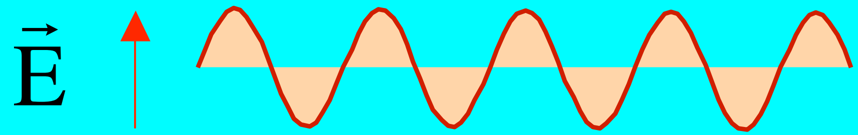

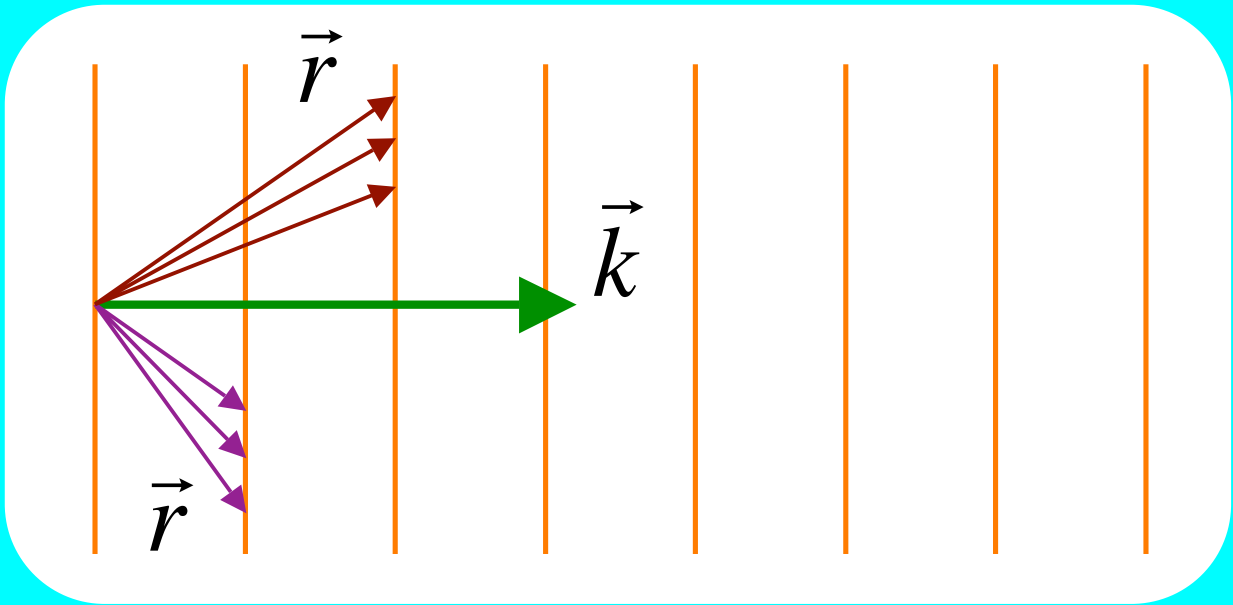

$$k = \frac{2\pi}{\lambda}$$

$$\vec{E}(\vec{r}) = \vec{E}_0 \cos(\vec{k}\cdot\vec{r}) \quad = \vec{E}_0 \quad \text{for} \quad \vec{k}\cdot\vec{r} = 2n\pi$$

$$\mathrm{Re}\left\{ e^{i\vec{k}\cdot\vec{r}} \right\}$$

Complex notation

Electromagnetic spectrum

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Photon energy

Wavelength

$$E = h\nu = hc/\lambda$$

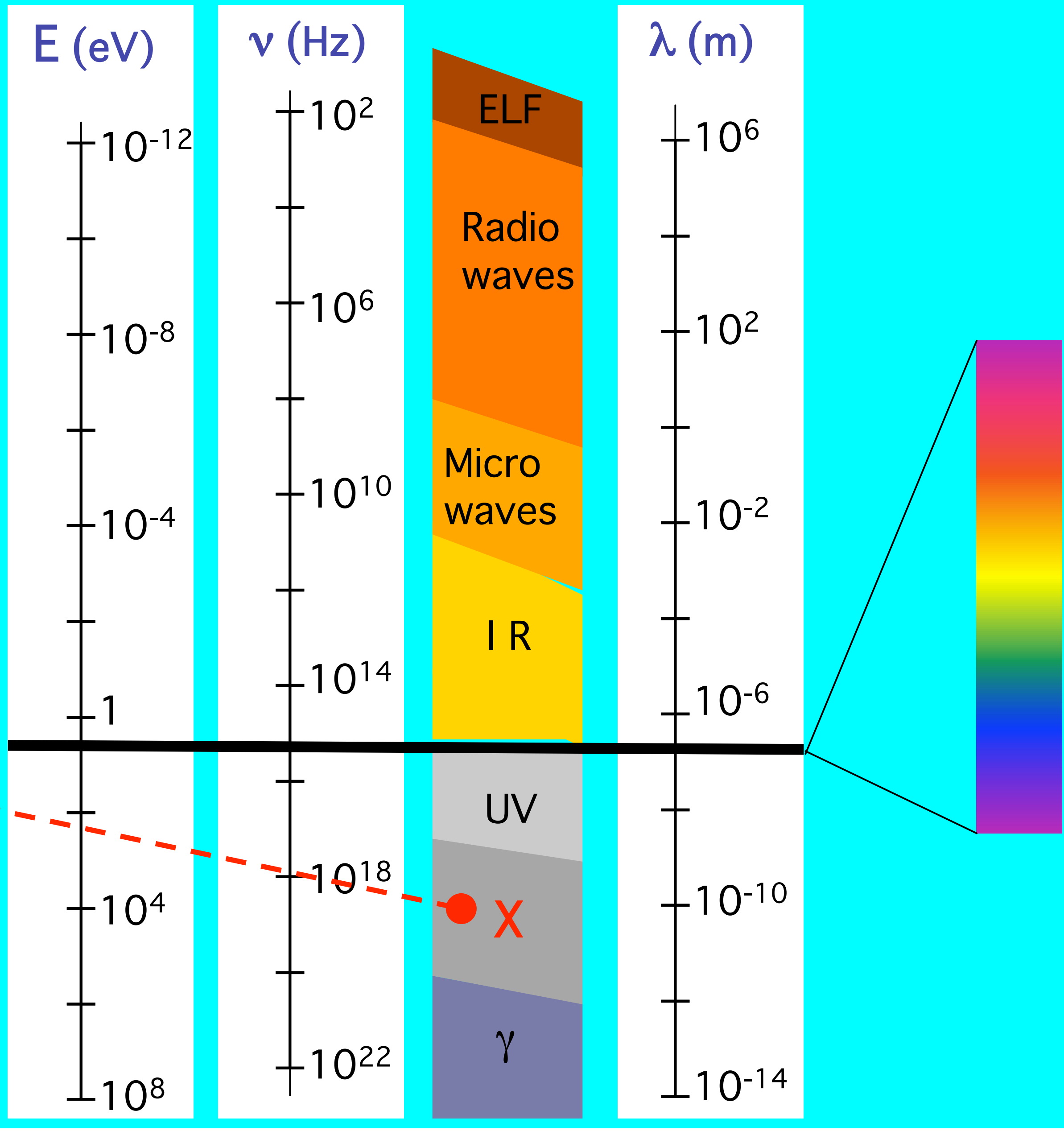
Frequency

X-rays

$\lambda \approx 0.01 \div 10$ Å

$\nu \approx 10^{17} \div 10^{20}$ Hz

$E \approx 0.4 \div 400$ keV



Interaction of x-rays with matter

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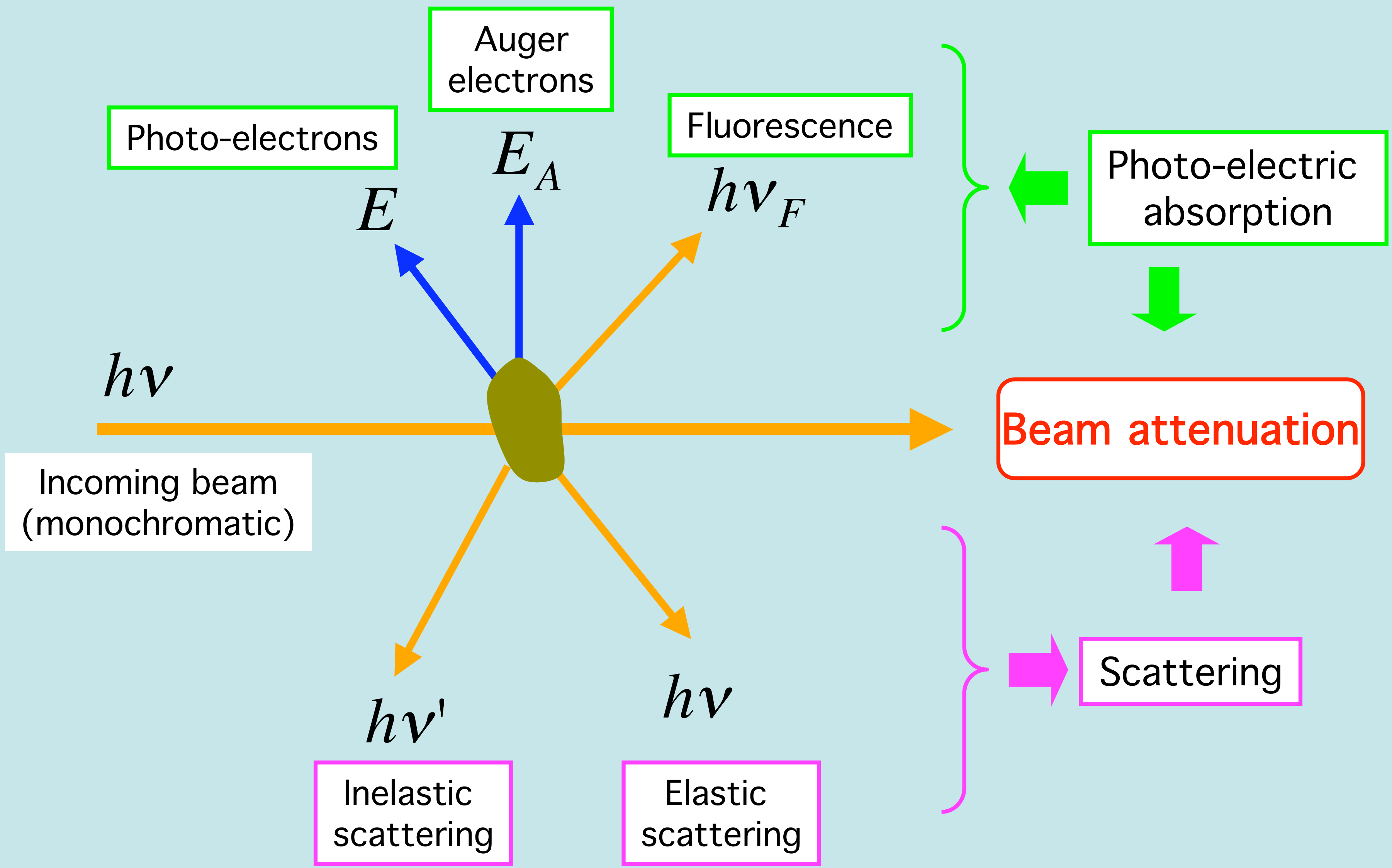

X-RAYS and X-ray techniques

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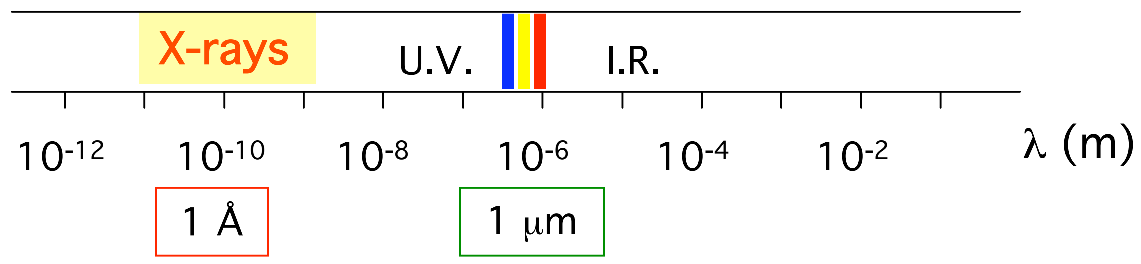
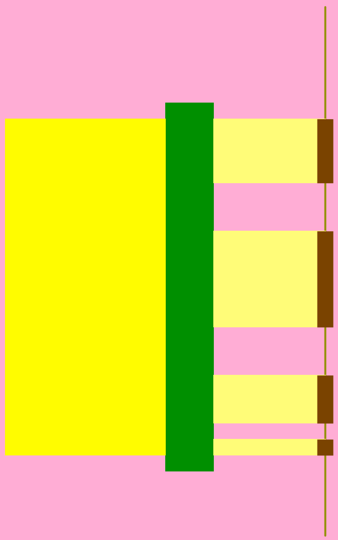
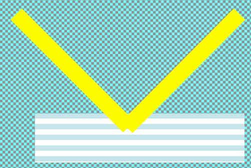


$$E[\mathrm{keV}] = \frac{12.4}{\lambda[\mathrm{\AA}]}$$

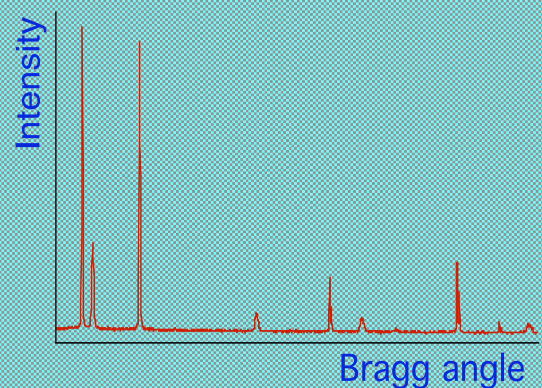
Imaging



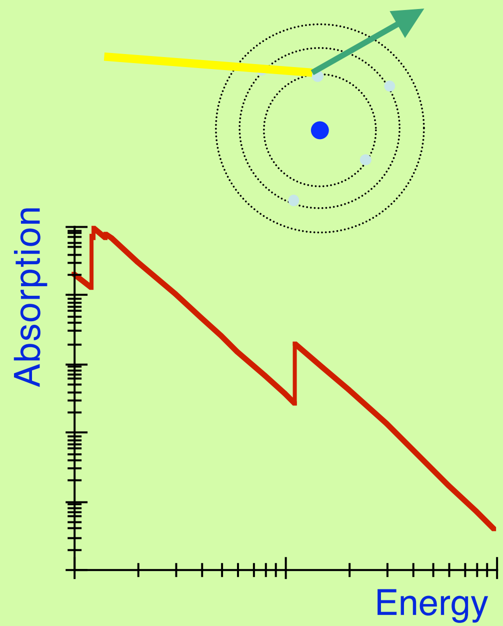
Scattering



$$\lambda = 2d\sin\theta$$



Spectroscopy



Crystals

Crystals

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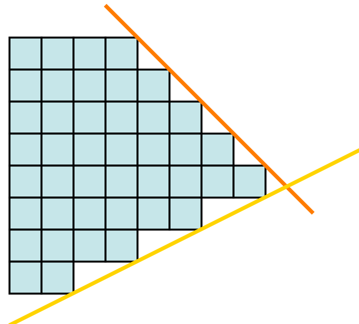
Quartz crystal (SiO_2)

Macroscopic regularities
(e.g. constancy of angles)

Classification of crystals



Regular packing
of microscopic structural units
R.J. Haüy (1743-1822)

Atoms and crystals

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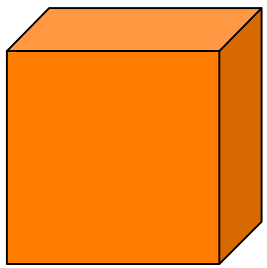
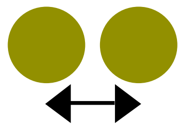
HYPOTHESIS: Structural units = atoms

Example: NaCl



Atomic masses: Na 38.12×10^{-24} g
Cl 58.85×10^{-24} g

Cubic structure
1 cm^3 m = 2.165 g
N = 44.6×10^{21} atoms



0.28 nm = 2.8 Å

CONCLUSION: Inter-atomic distances
Atomic dimensions
≈ X-ray wavelengths

X-ray diffraction from crystals

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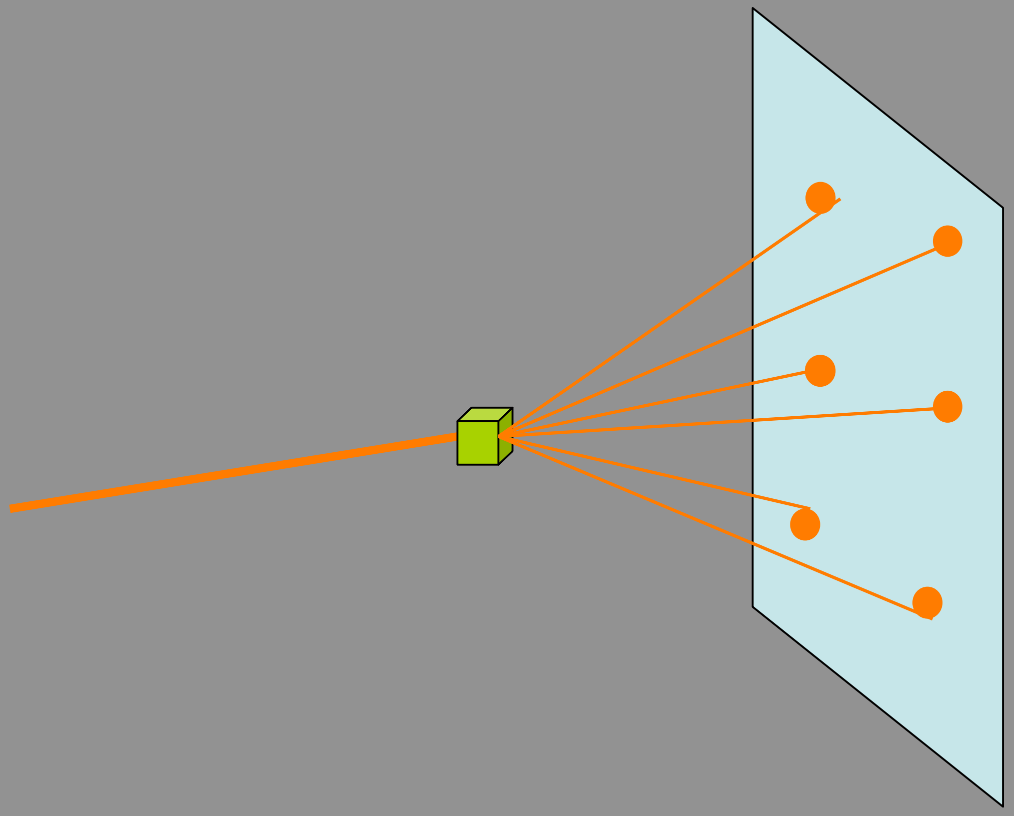
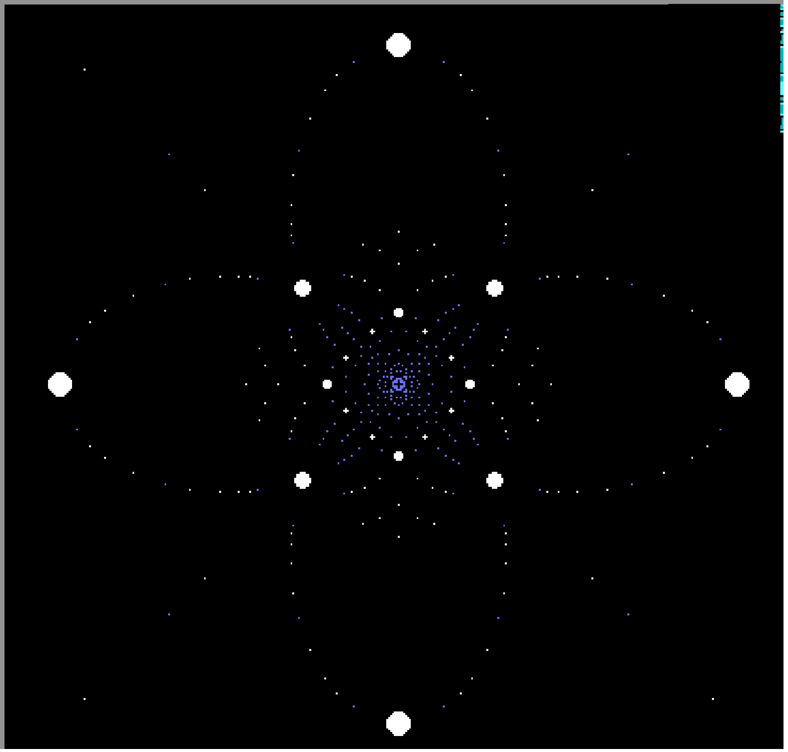
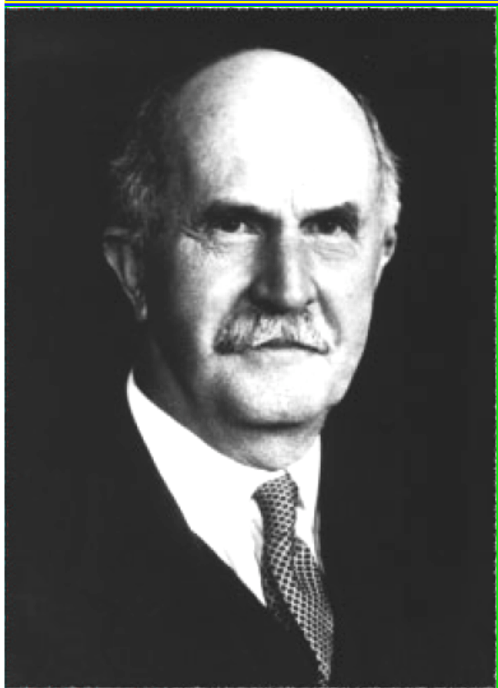


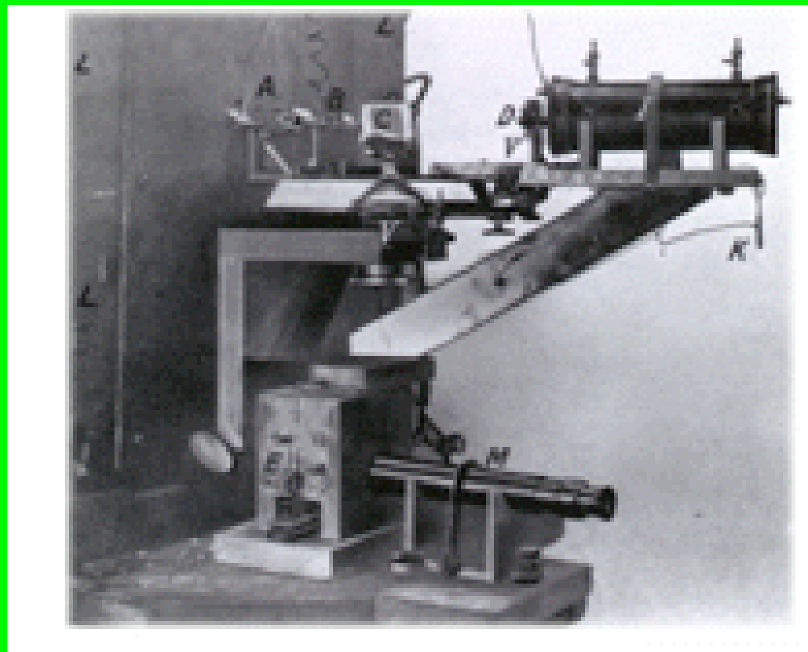


Munich, 1912:
- Max von Laue
- W. Friedrich & P.Knipping

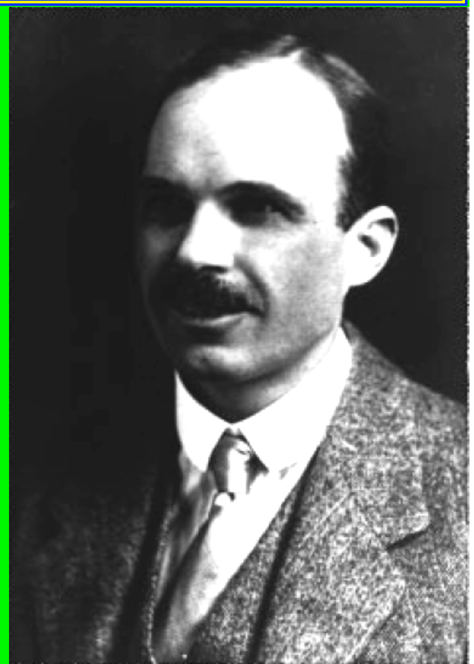
Cambridge, 1912/13



William Henry Bragg
(1862-1942)



Bragg spectrometer



William Lawrence Bragg
(1890-1971)

Crystal structure

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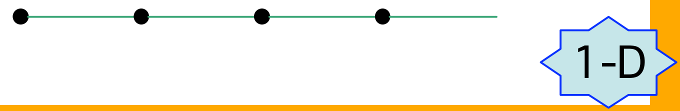
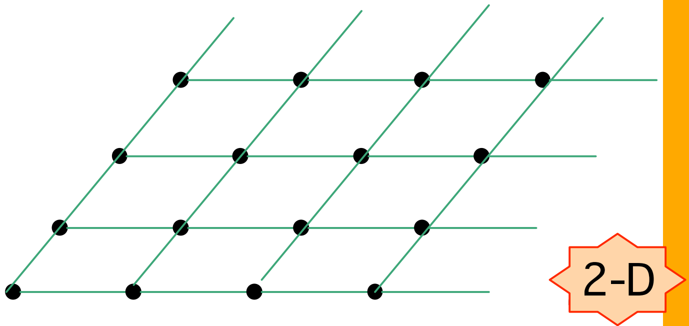
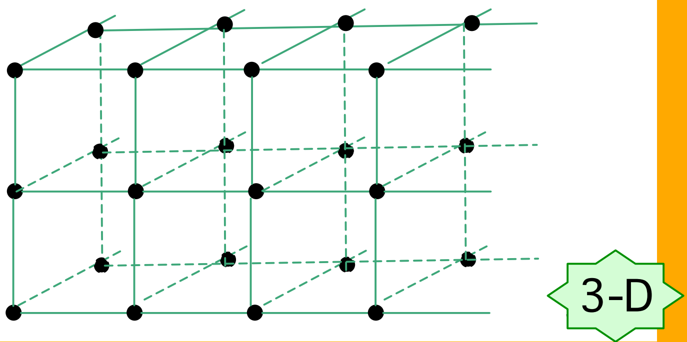
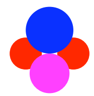
Bravais lattice + Basis



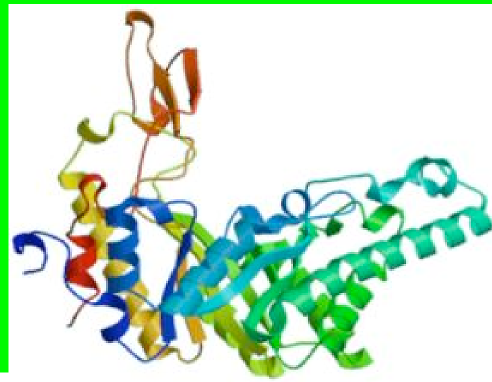




Atom



Molecule



Protein

Bravais lattice + basis

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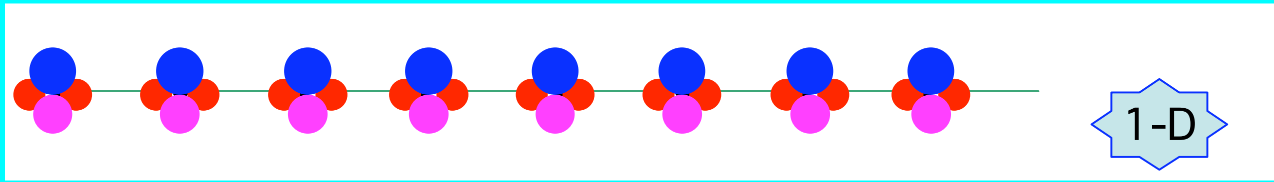

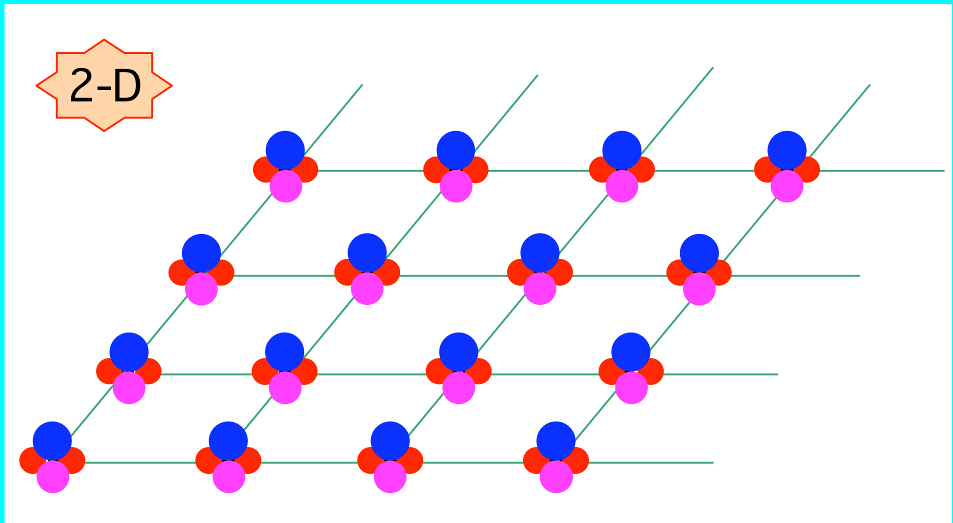

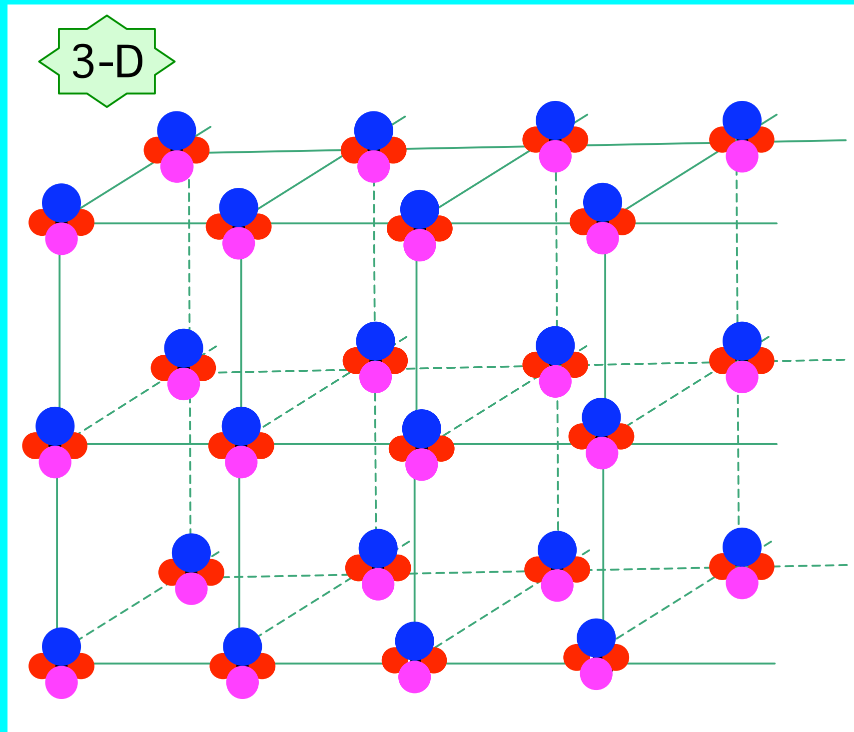

Crystal lattices

Translation vectors (2D)

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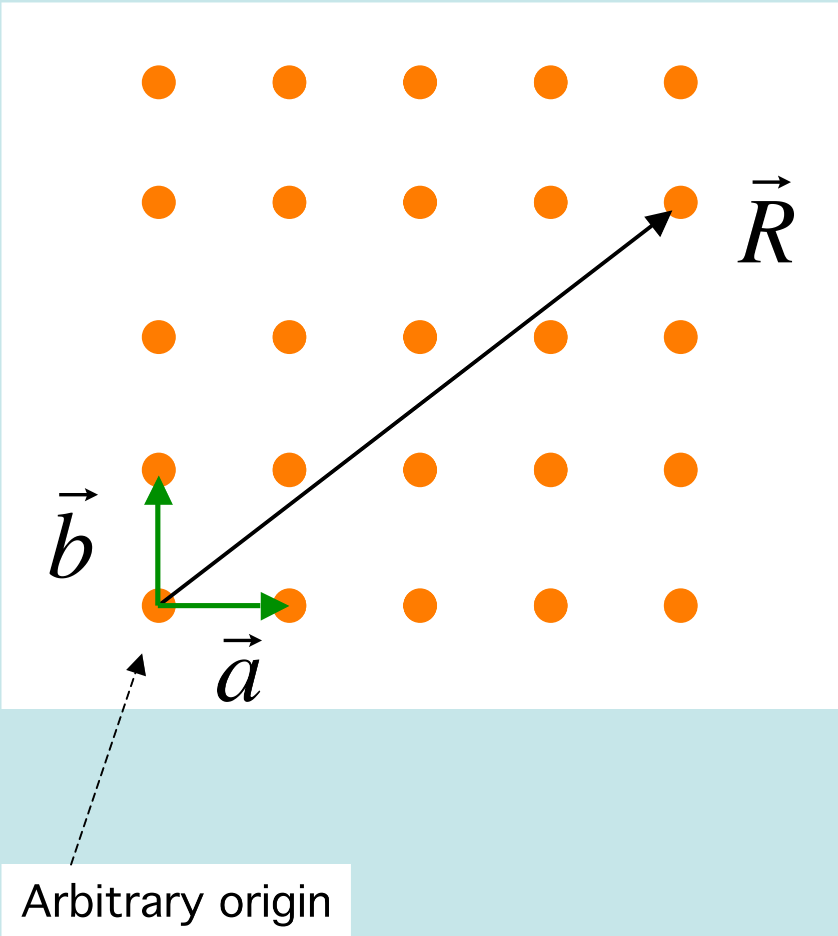
2-D



For every lattice point

$$\vec{R} = n_1\vec{a} + n_2\vec{b}$$

integers

primitive vectors

Primitive vectors (2D)

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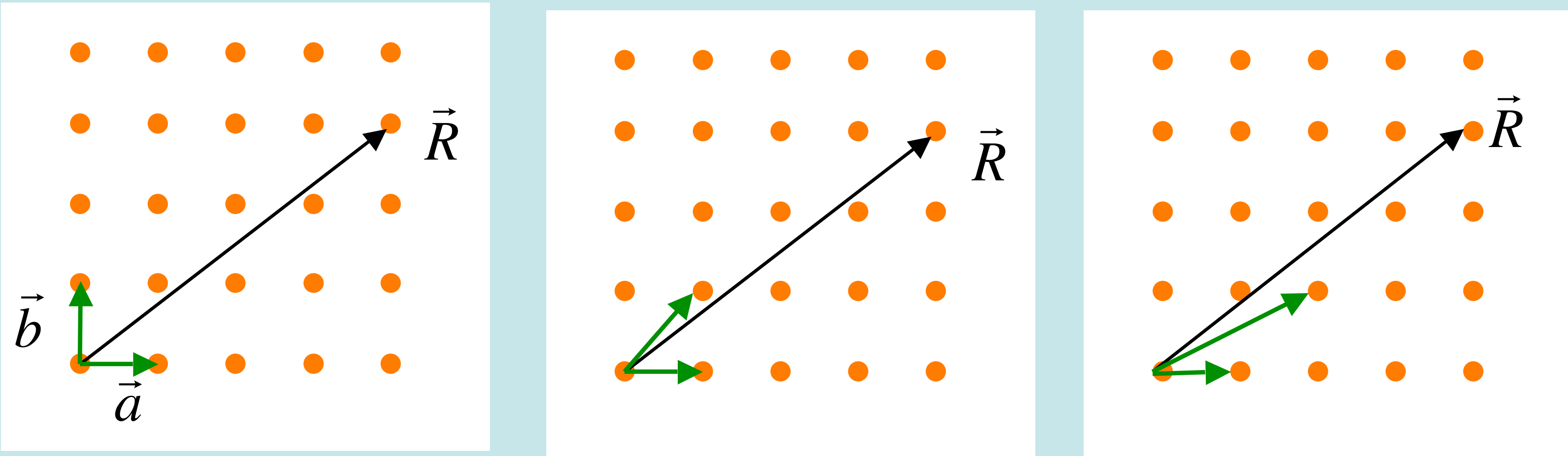
2-D

$$\vec{R} = n_1\vec{a} + n_2\vec{b}$$



Different choices of primitive vectors $\vec{a},\vec{b}$

Non-primitive vectors (2D)

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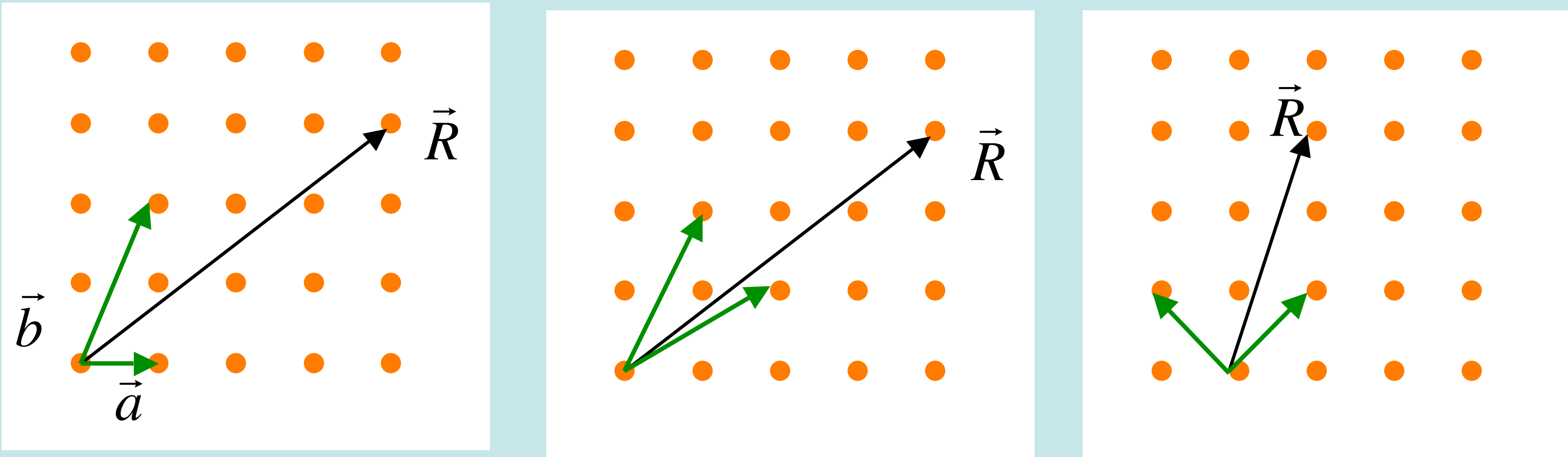
2-D

Not all $\vec{a},\vec{b}$ pairs are primitive



$$\vec{R} \neq n_1\vec{a} + n_2\vec{b}$$

Primitive vectors (3D)

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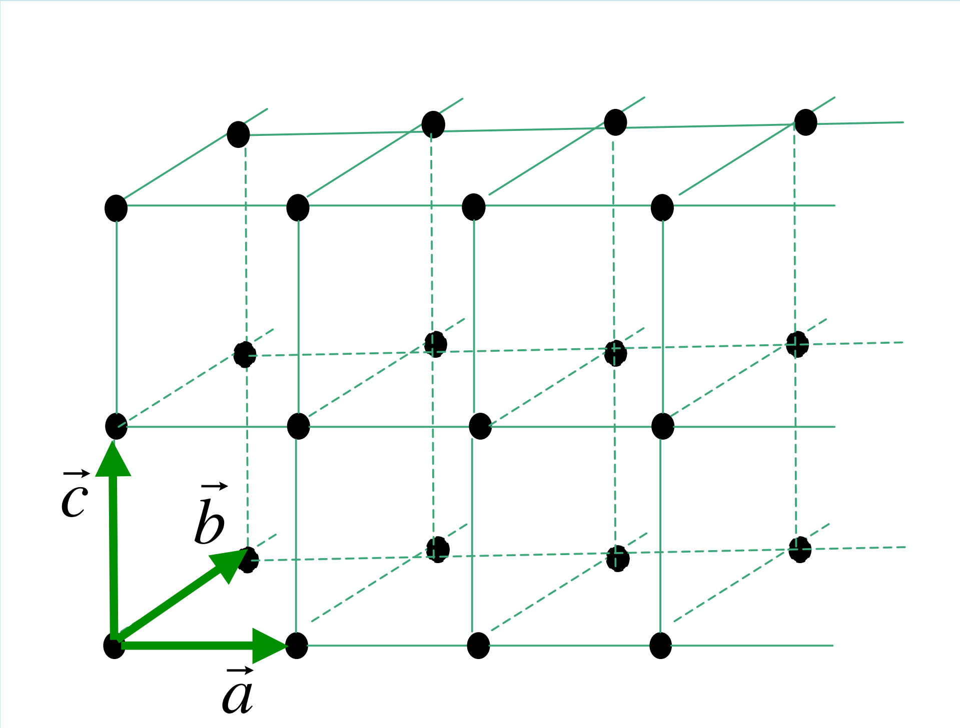
$$\vec{R} = n_1\vec{a} + n_2\vec{b} + n_3\vec{c}$$

3-D



Different choices of primitive vectors $\vec{a}, \vec{b}, \vec{c}$

Primitive unit cells (2D)

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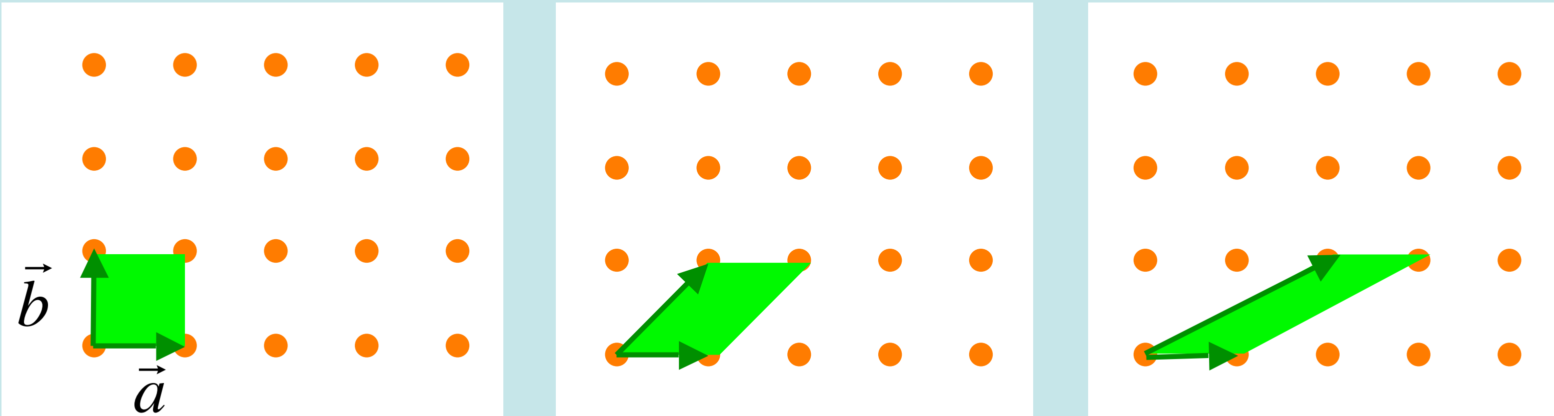
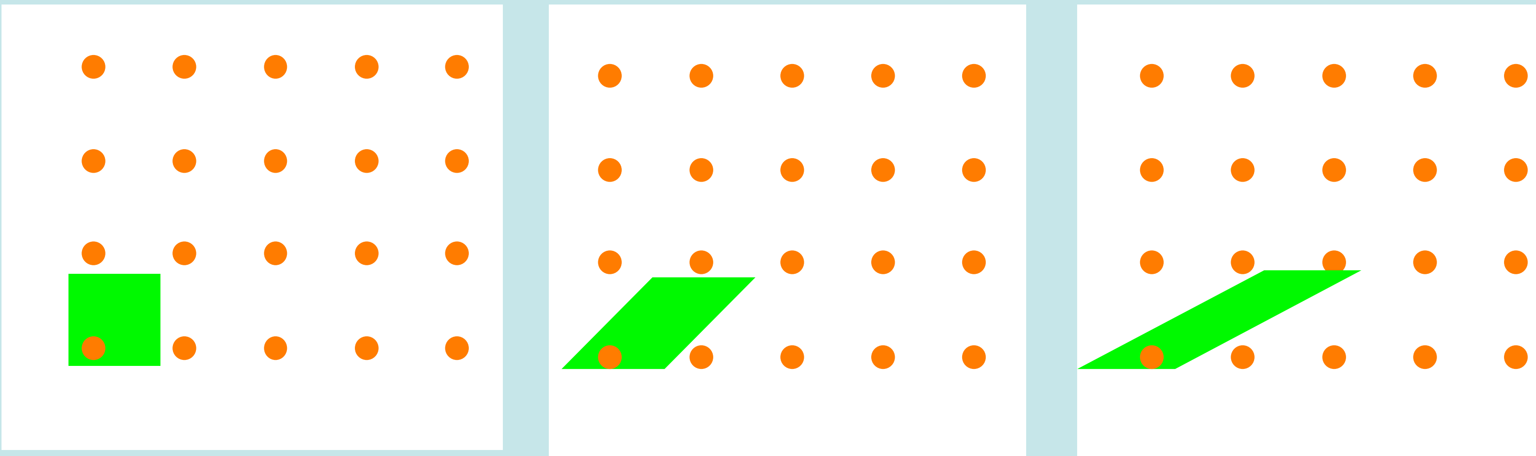
2-D



Different choices of primitive unit cells



Primitive cell = 1 lattice point

Conventional unit cells (2D)

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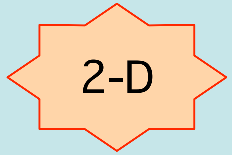

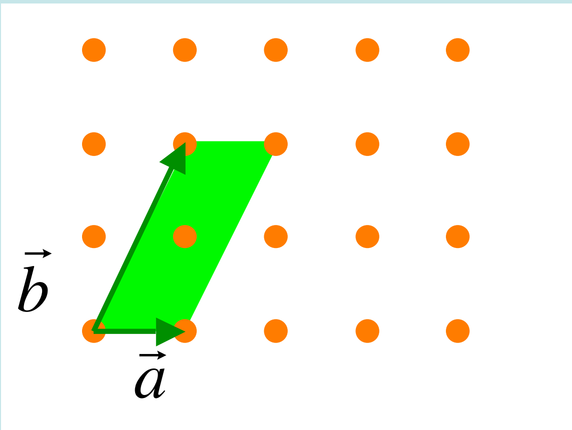

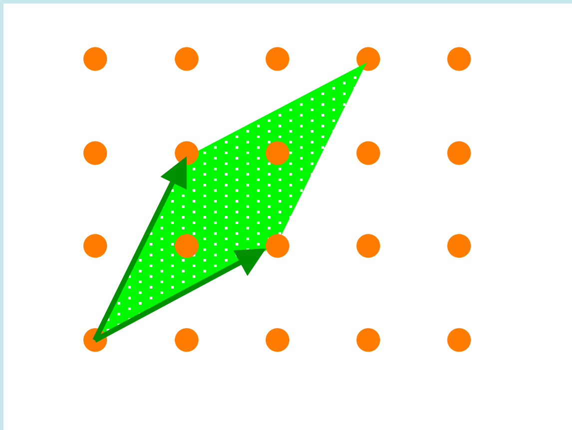

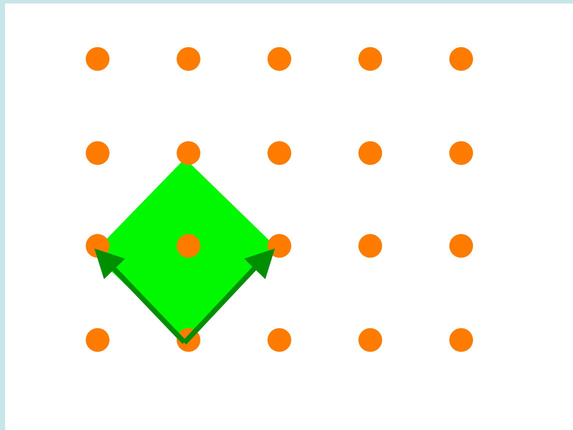

More than 1 lattice point per unit cell

Non-Bravais lattices

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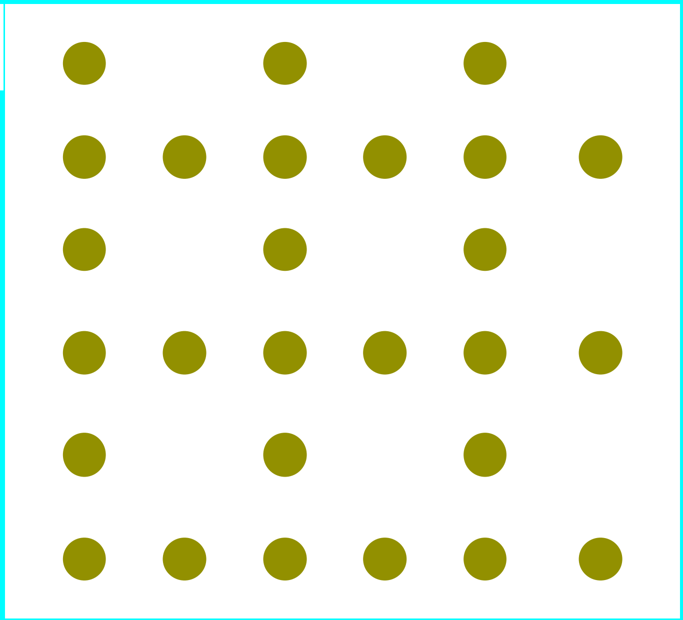
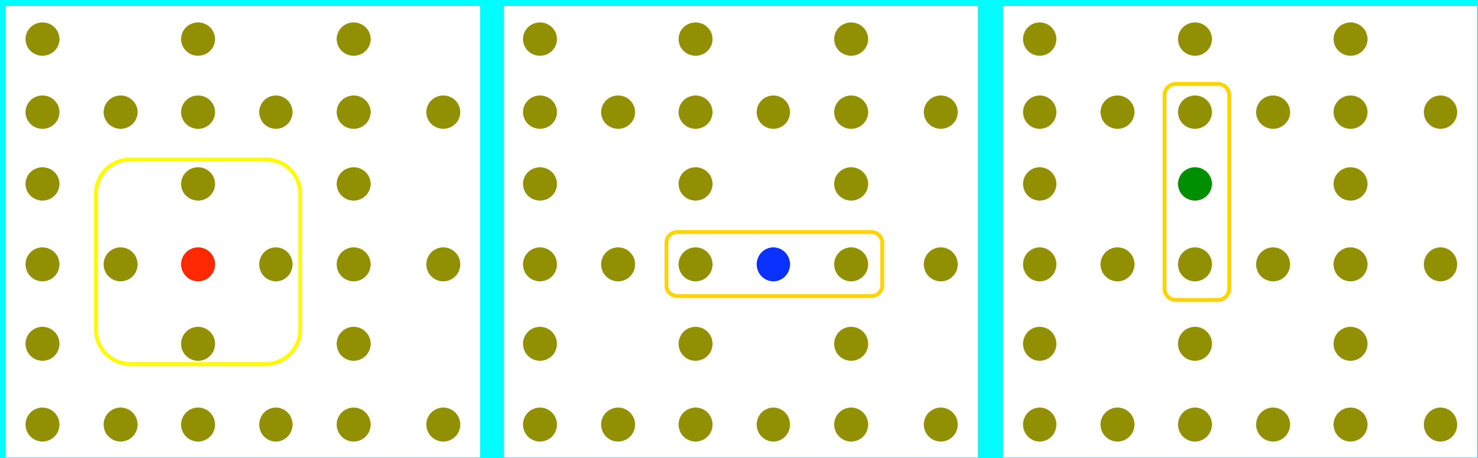
2-D

Atoms



$$\vec{R} \neq n_1\vec{a} + n_2\vec{b}$$

Un-equivalent sites



Bravais lattices

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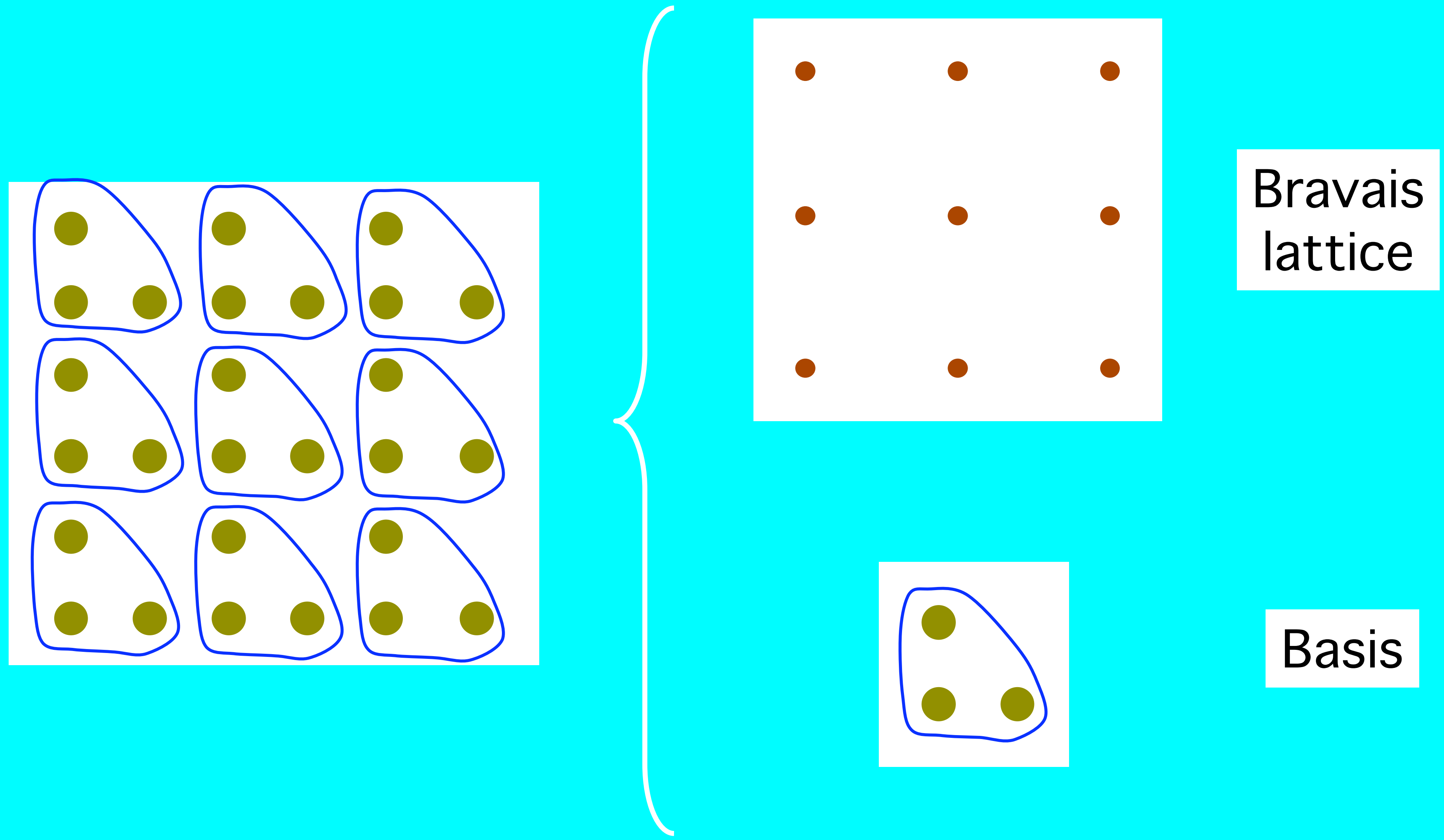
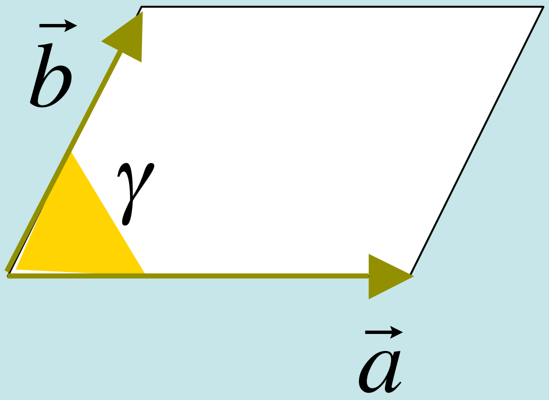
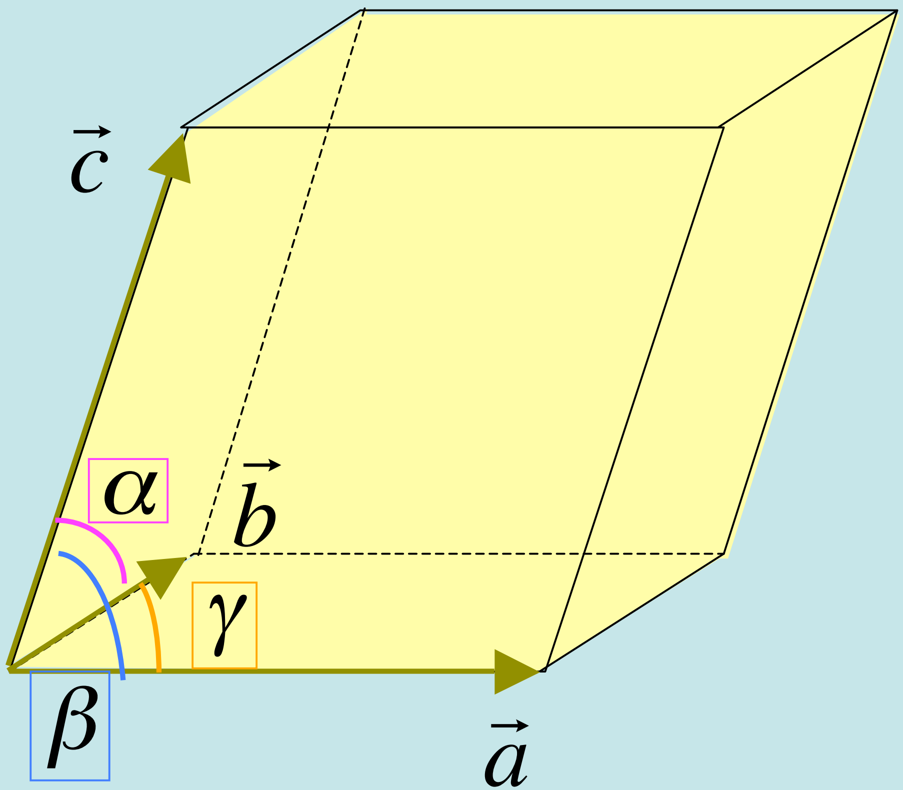


Classification of cells

2-D



3-D



a	b	c	latin
α	β	γ	greek

Surface Bravais lattice

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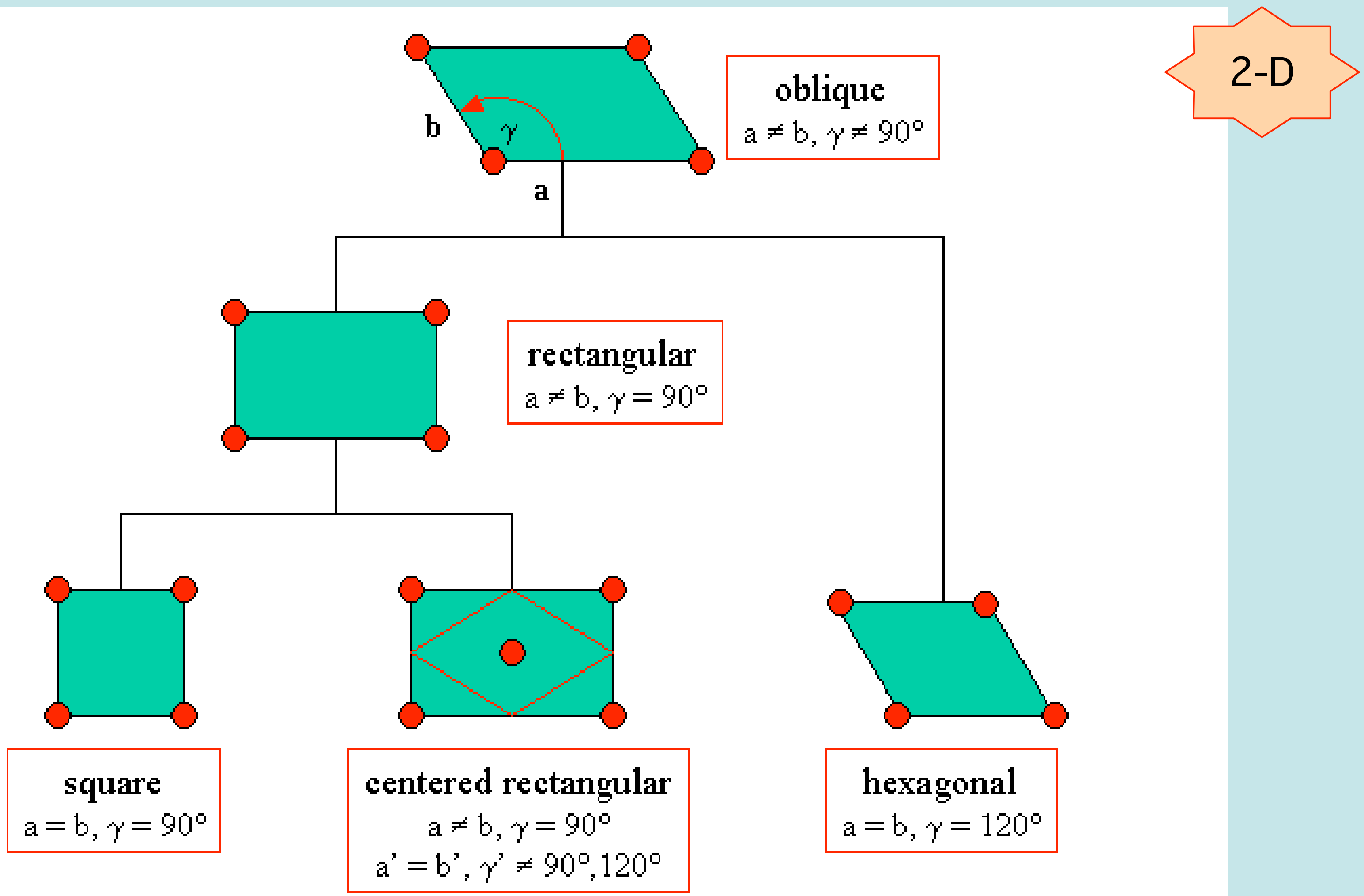

Non-primitive unit cell

3-D Bravais lattices

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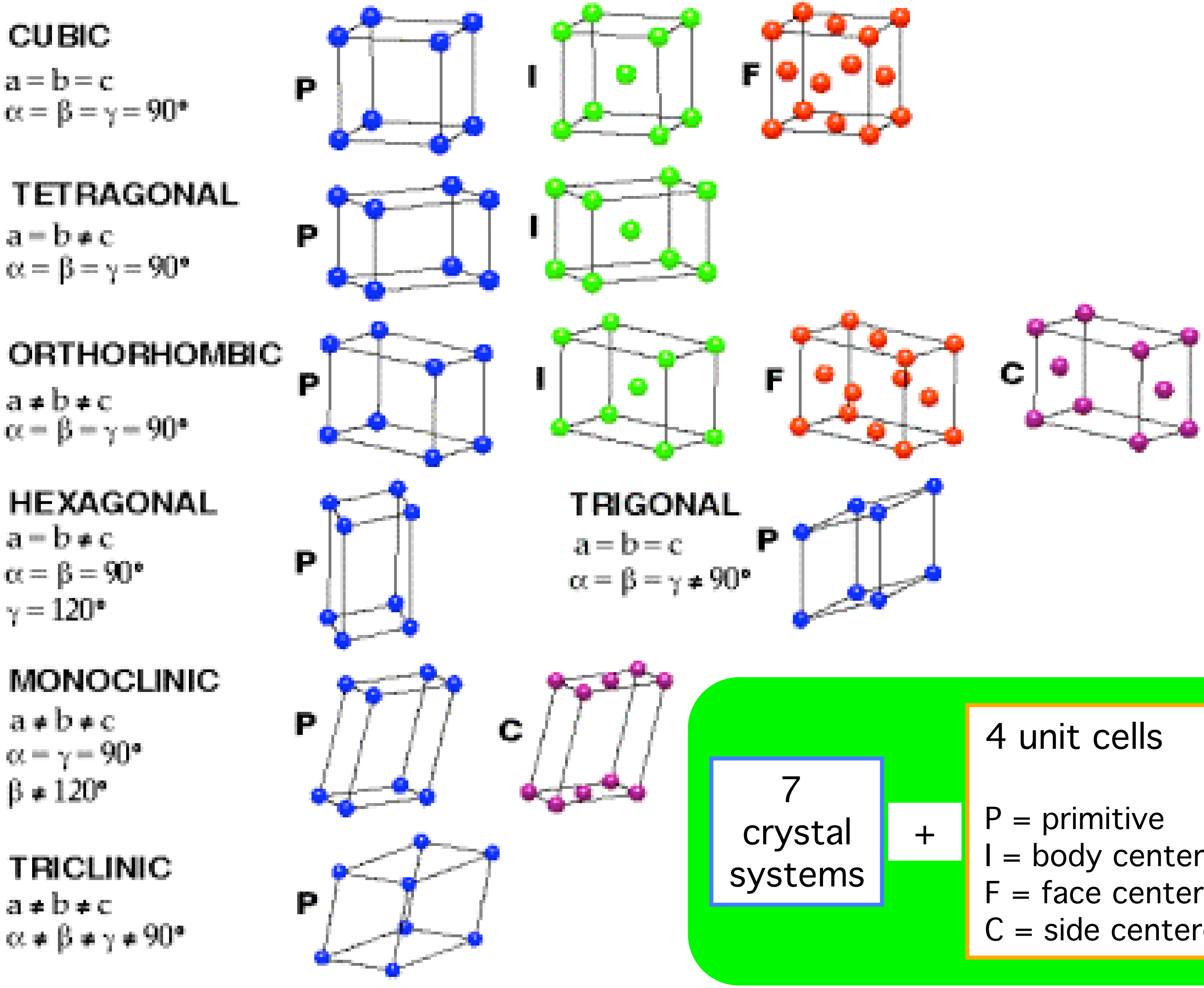
3-D



7 crystal systems + 4 unit cells = 14 Bravais lattices

4 unit cells:

P = primitive
I = body centered
F = face centered
C = side centered

Coordinates

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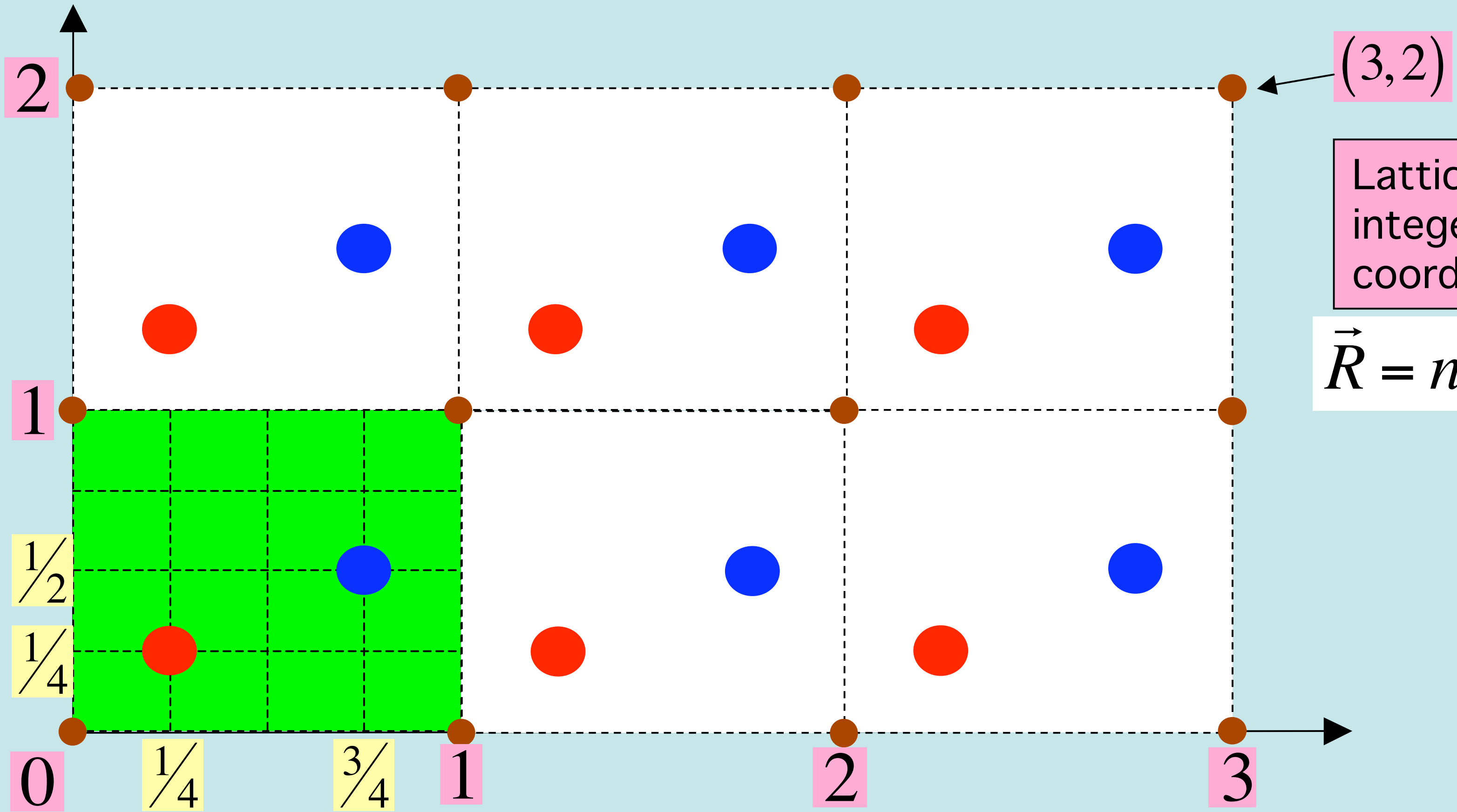

Lattice points: integer coordinates

$$\vec{R} = n_1\vec{a} + n_2\vec{b}$$

Inside cell: fractional coordinates

$\left(\frac{1}{4},\frac{1}{4}\right)$ $\left(\frac{3}{4},\frac{1}{2}\right)$

- Some relevant crystal structures

Simple cubic lattice

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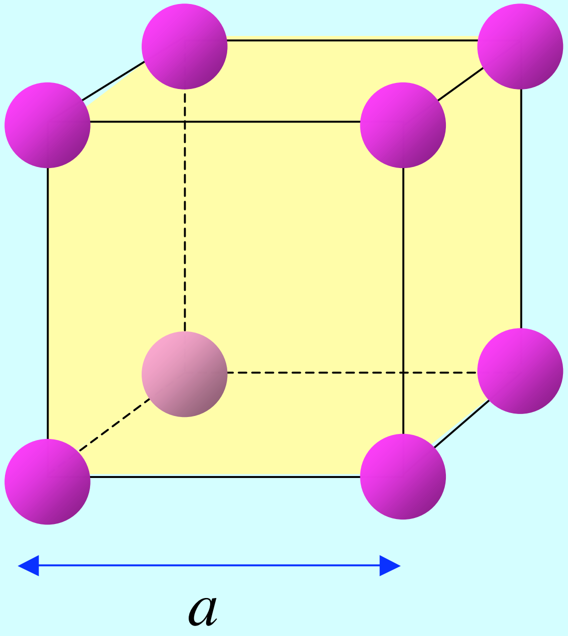
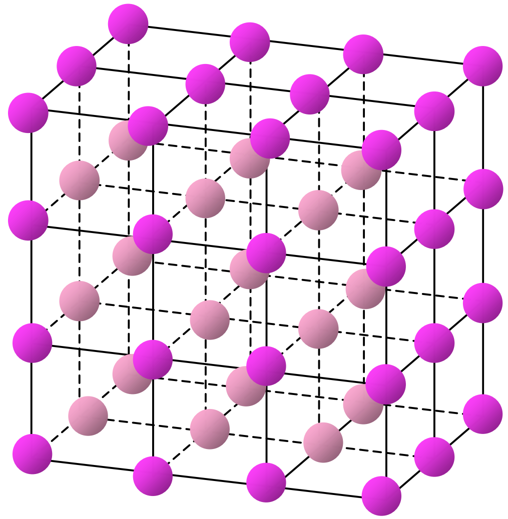
84-Po   a=3.35 Å

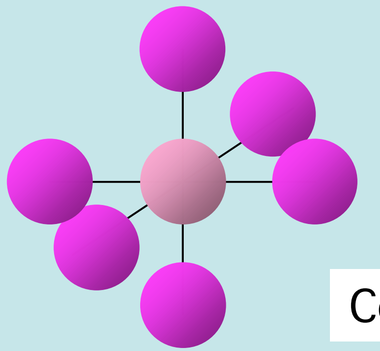


lattice parameter

Primitive unit cell
(1 lattice point per cell)

Bravais lattice





Coordination number = 6

Body centered cubic lattice (bcc)

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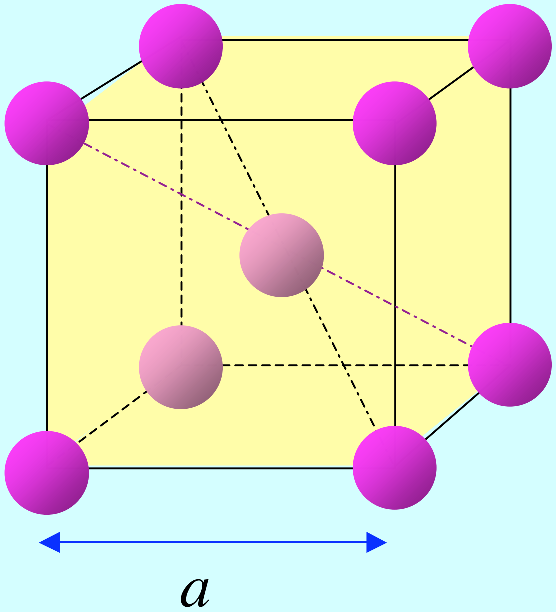
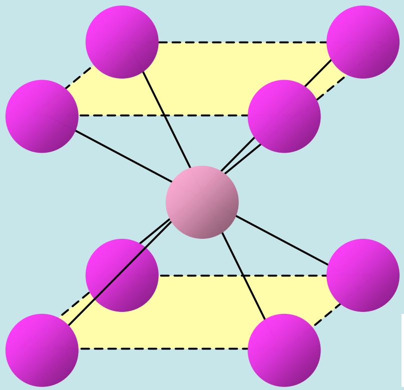


lattice parameter

conventional unit cell
(2 lattice points per cell)

24-Cr	a=2.88 Å
26-Fe	a=2.87 Å
42-Mo	a=3.15 Å

Bravais lattice



Coordination number = 8

Face centered cubic lattice (fcc)

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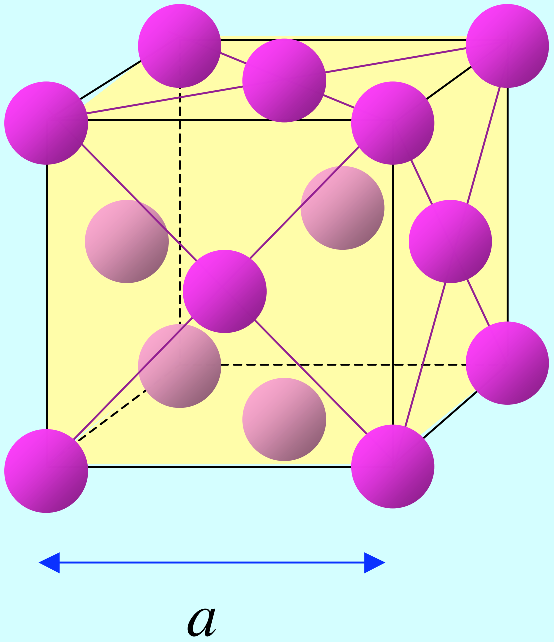
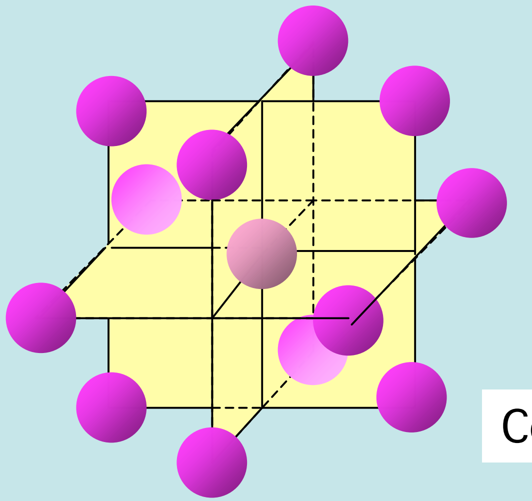


lattice parameter

conventional unit cell
(4 lattice points per cell)

29-Cu	a=3.61 Å
47-Ag	a=4.09 Å
79-Au	a=4.08 Å

Bravais lattice



Coordination number = 12

Diamond structure

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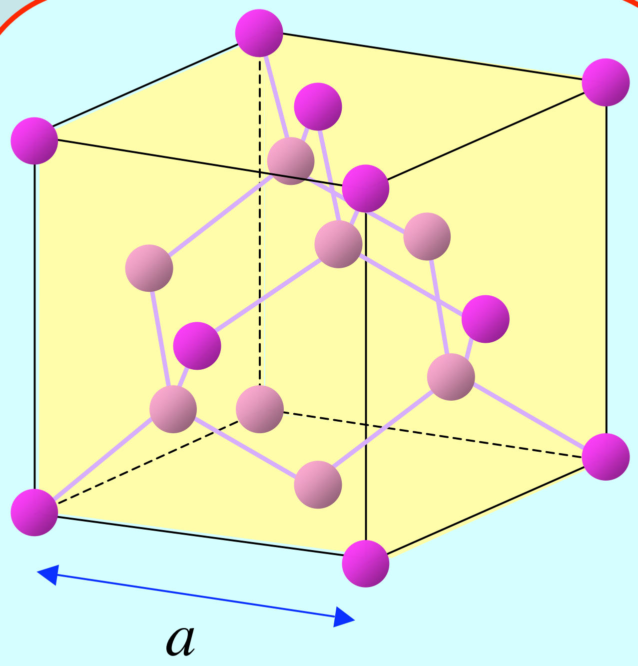
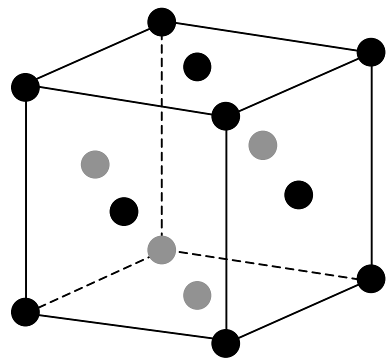
6-C	a=3.57 Å
14-Si	a=5.43 Å
32-Ge	a=5.66 Å

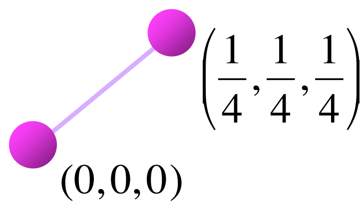


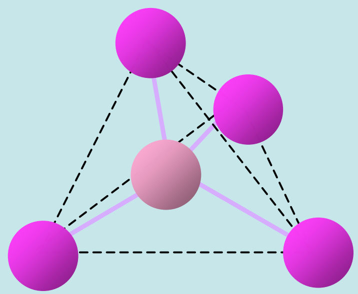
conventional unit cell
(8 atoms per cell)

Non-Bravais lattice

fcc Bravais lattice + 2-atom basis







Coordination number = 4

Zincblende (sphalerite) structure

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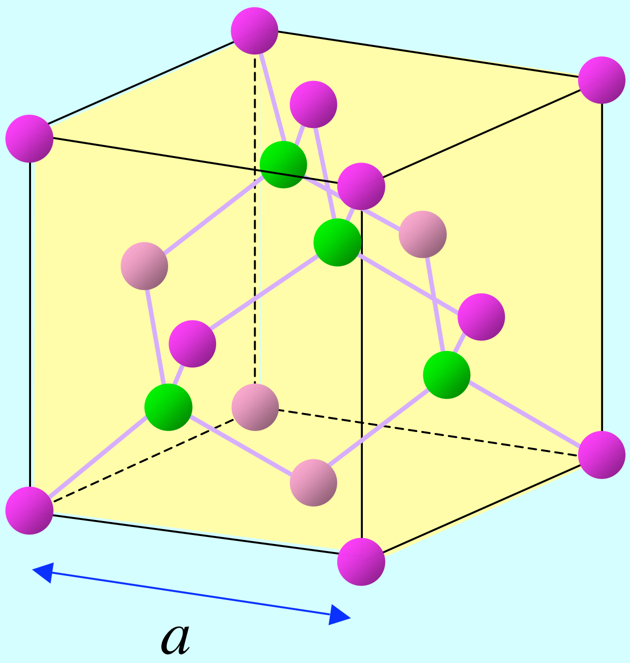
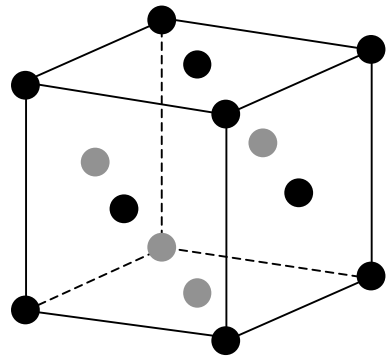
ZnS	a=5.41 Å
GaAs	a=5.65 Å
SiC	a=4.35 Å

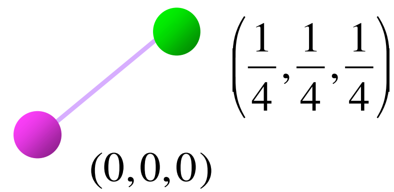


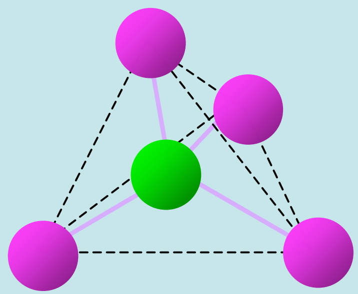
conventional unit cell
(8 atoms per cell)

Non-Bravais lattice

fcc Bravais lattice + 2-atom basis







Cordination number = 4

Rock-salt (NaCl) structure

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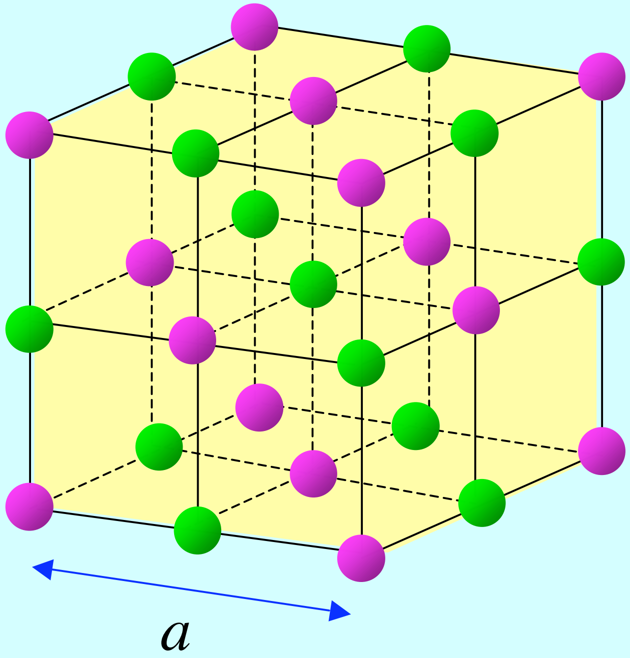
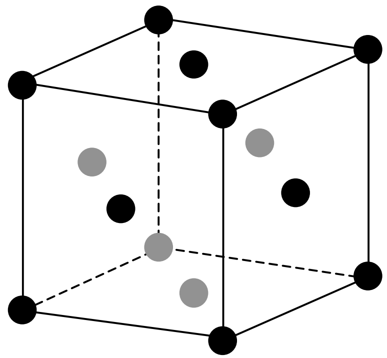
NaCl	a=5.64 Å
KBr	a=6.60 Å
CaO	a=4.81 Å

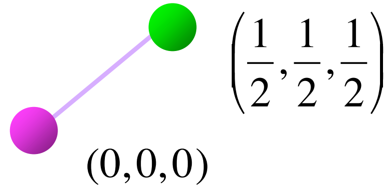


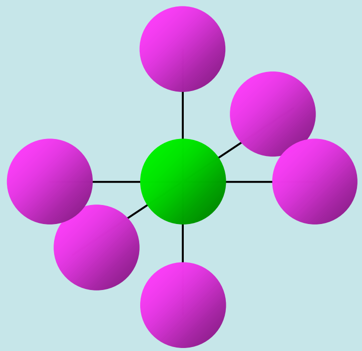
conventional unit cell
(8 atoms per cell)

Non-Bravais lattice

fcc Bravais lattice + 2-atom basis







Cordination number = 6

Simple hexagonal structure

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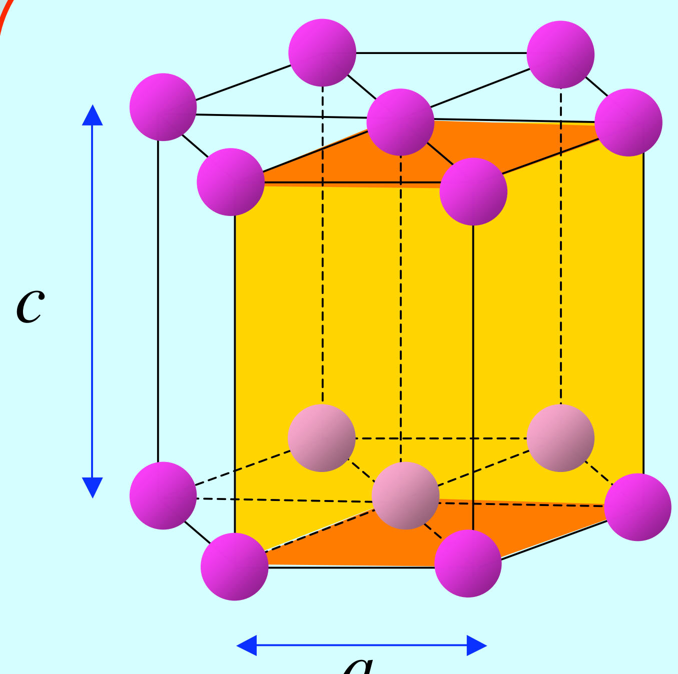
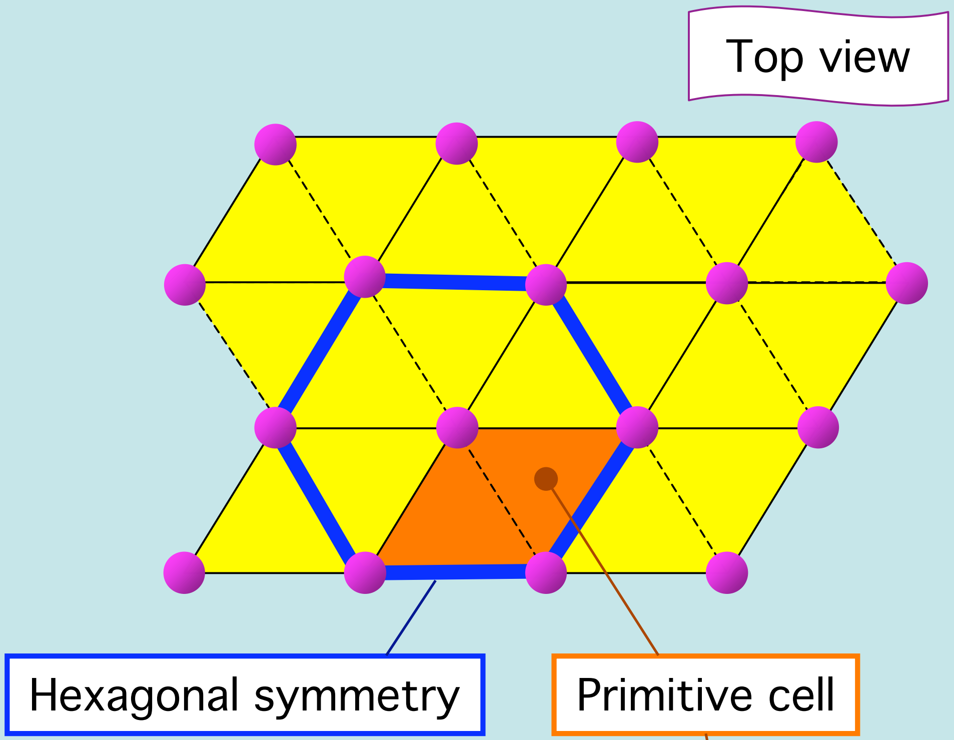
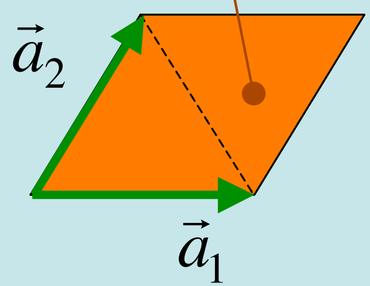


2 lattice parameters

Primitive unit cell
(1 lattice point per cell)



$$a_1 = a_2$$



Hexagonal close packed structure

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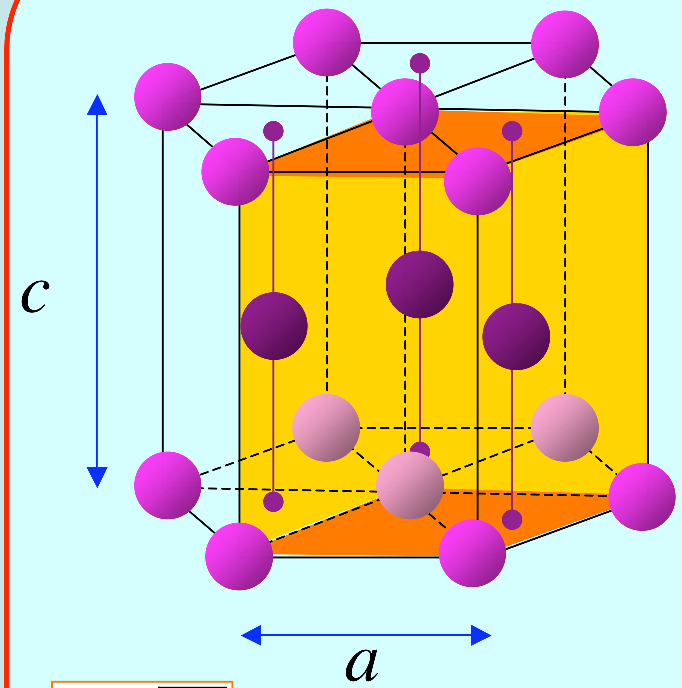


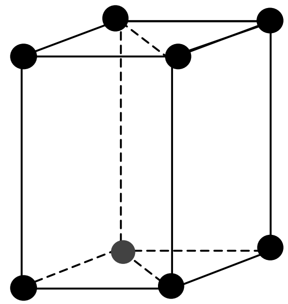
$$c = \sqrt{\frac{8}{3}}\,a$$

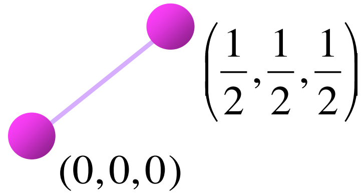
Conventional unit cell
(2 atoms per cell)

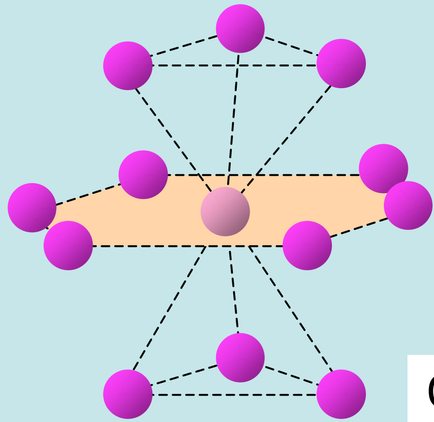
4-Be	a=2.29 Å
12-Mg	a=3.21 Å
48-Cd	a=2.98 Å

Non-Bravais lattice

primitive cell + 2-atom basis







Cordination number = 12

Coordination number

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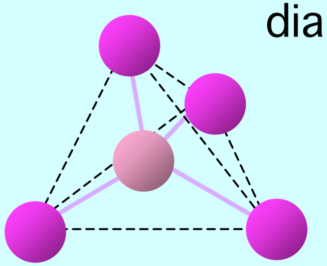
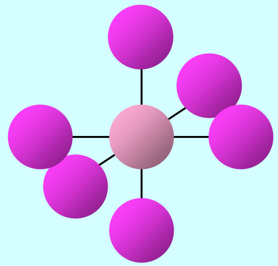
diamond



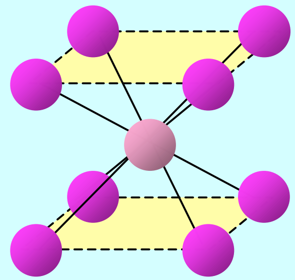
N=4

cubic



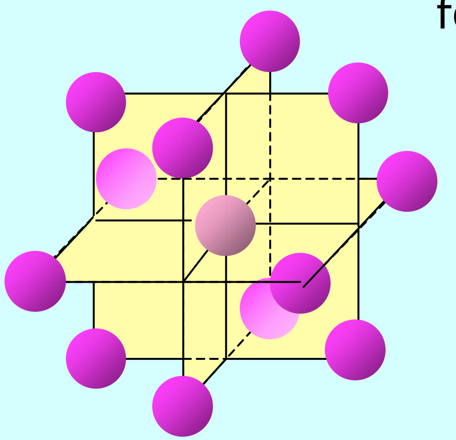
N=6

bcc

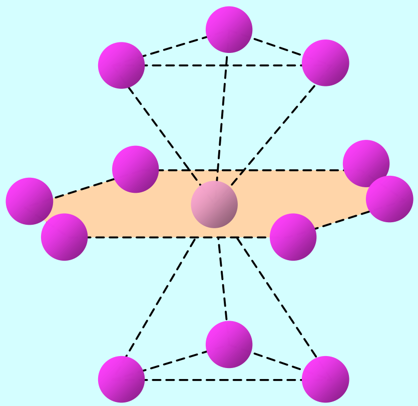


N=8

fcc



hcp



N=12

Close packing

Close-packing of spheres

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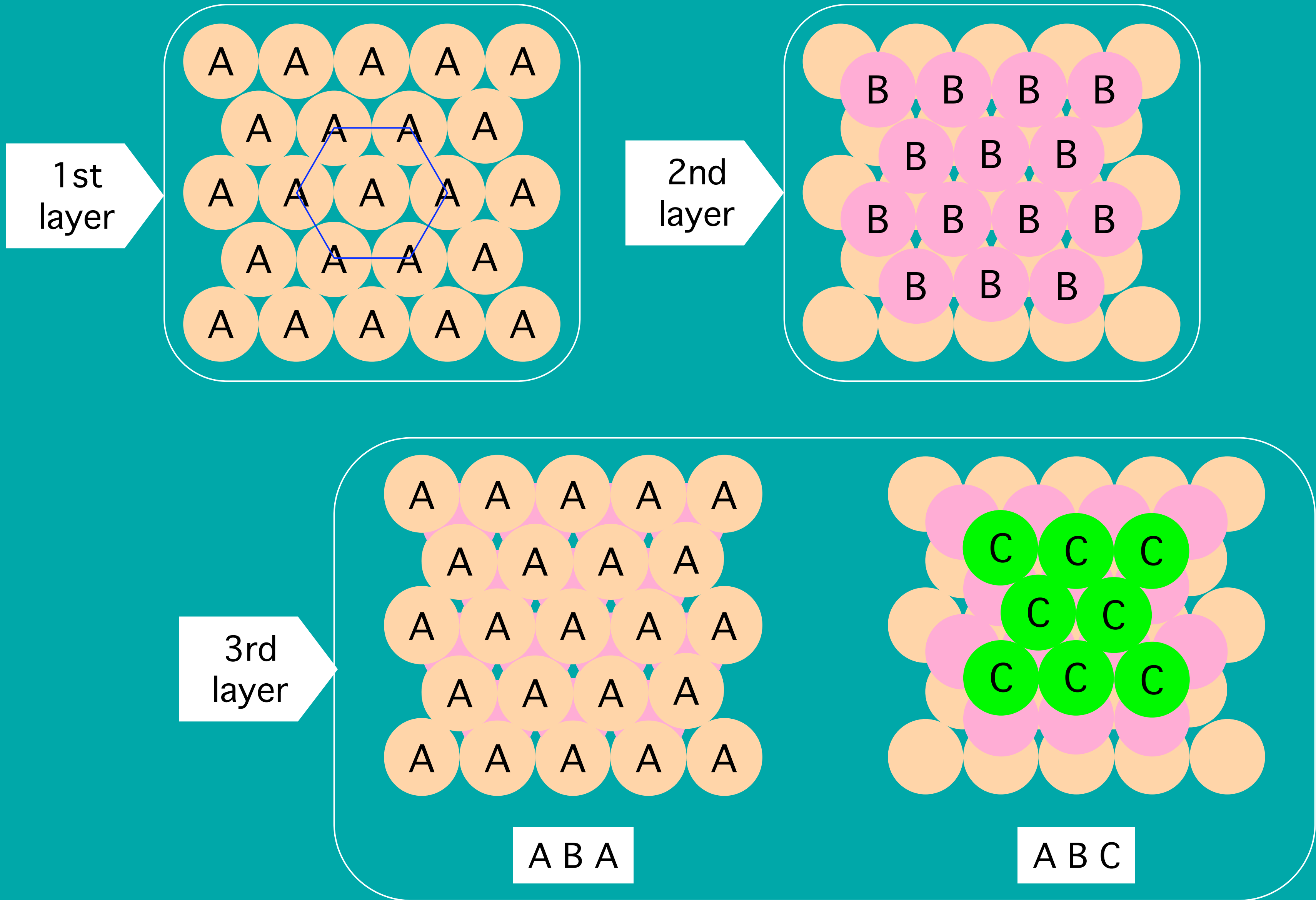

hcp versus fcc

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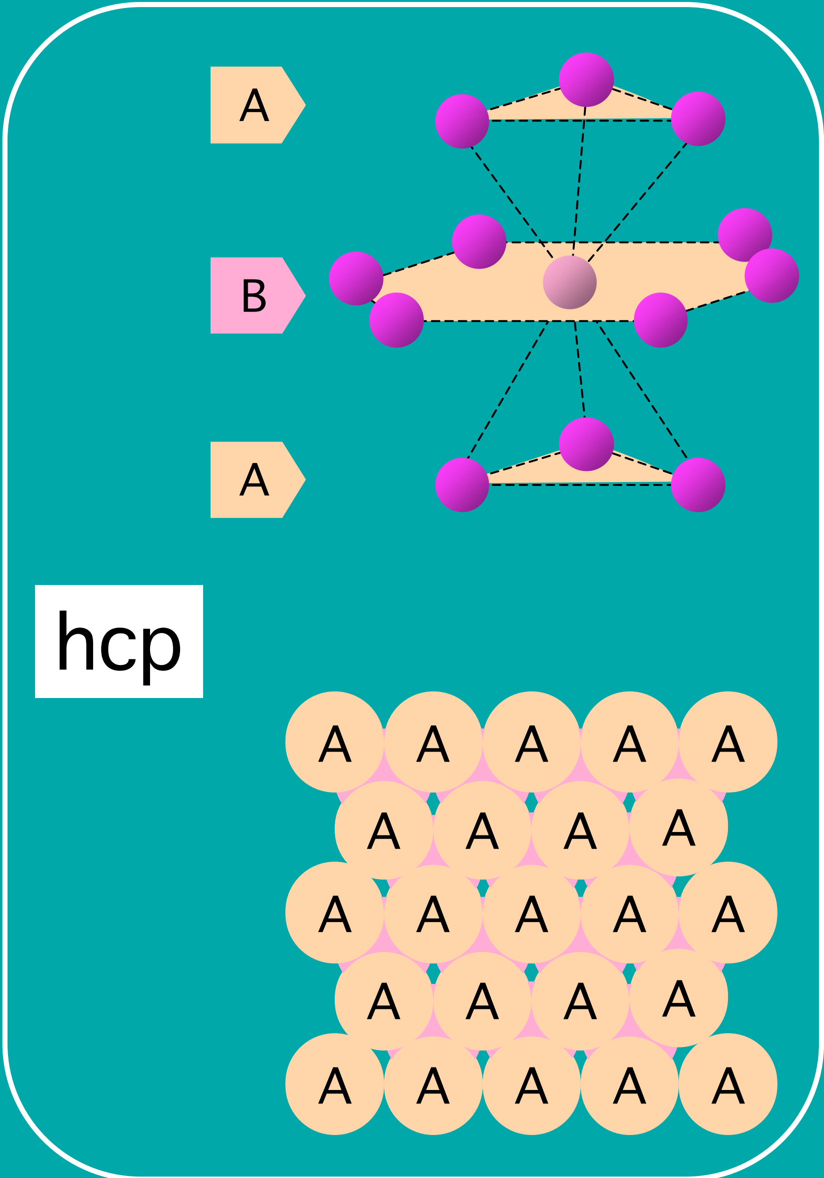

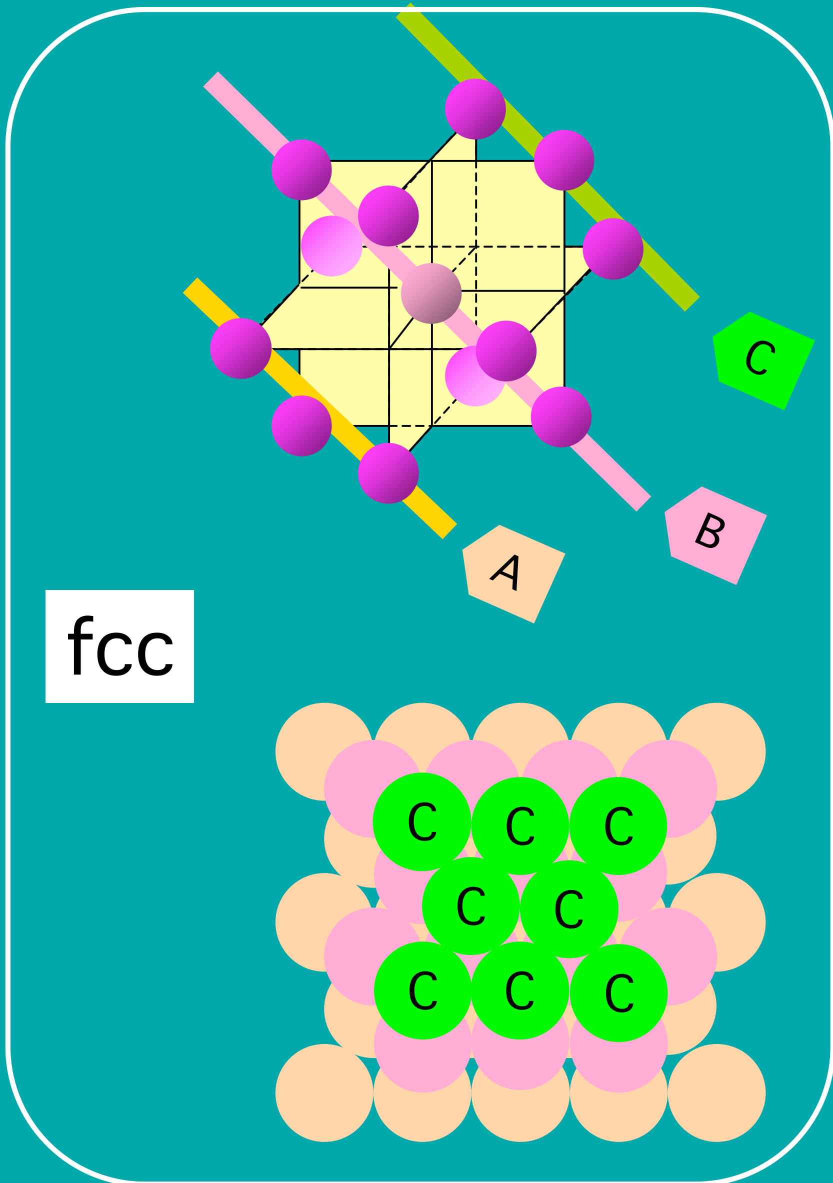

Crystal planes

Crystal planes

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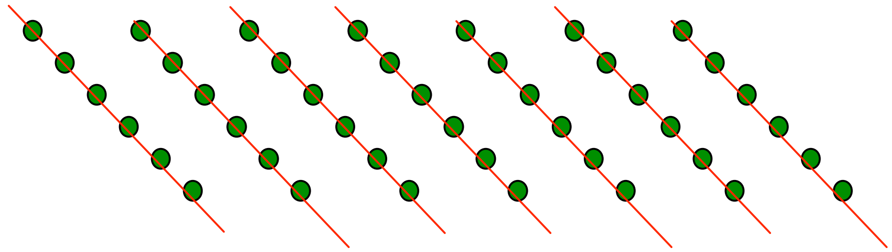
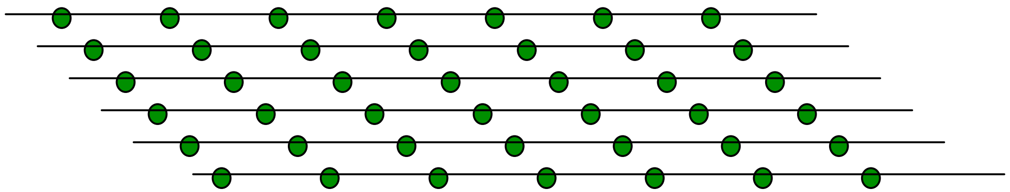
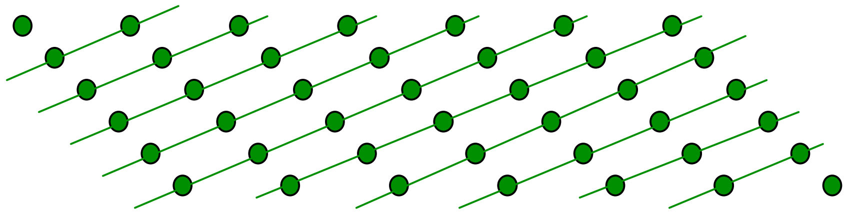
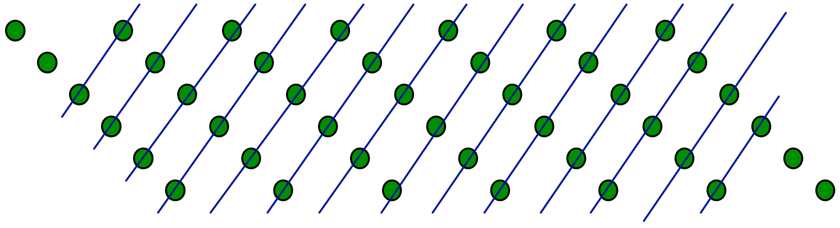
2-D









Miller indices, 2-D (a)

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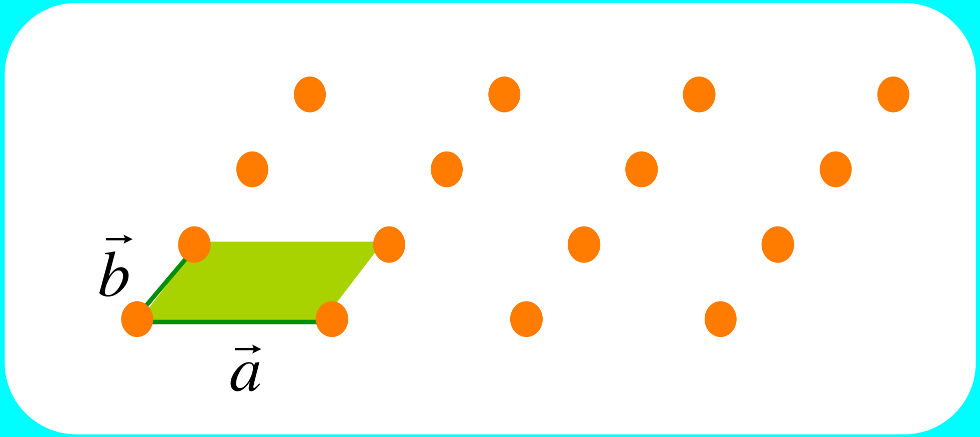
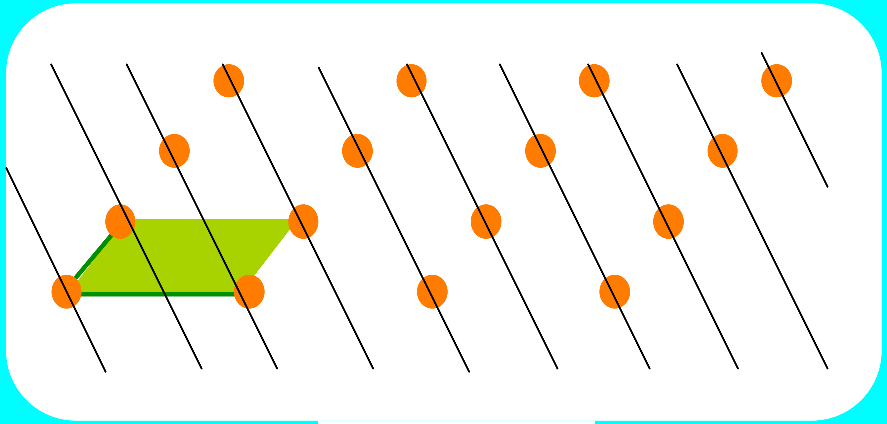
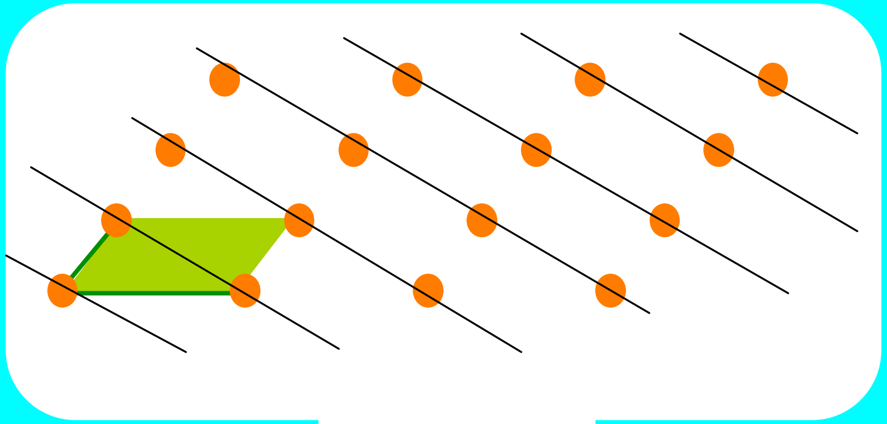
2-D





(hk) = (21)



(hk) = (11)

Miller indices, 2-D (b)

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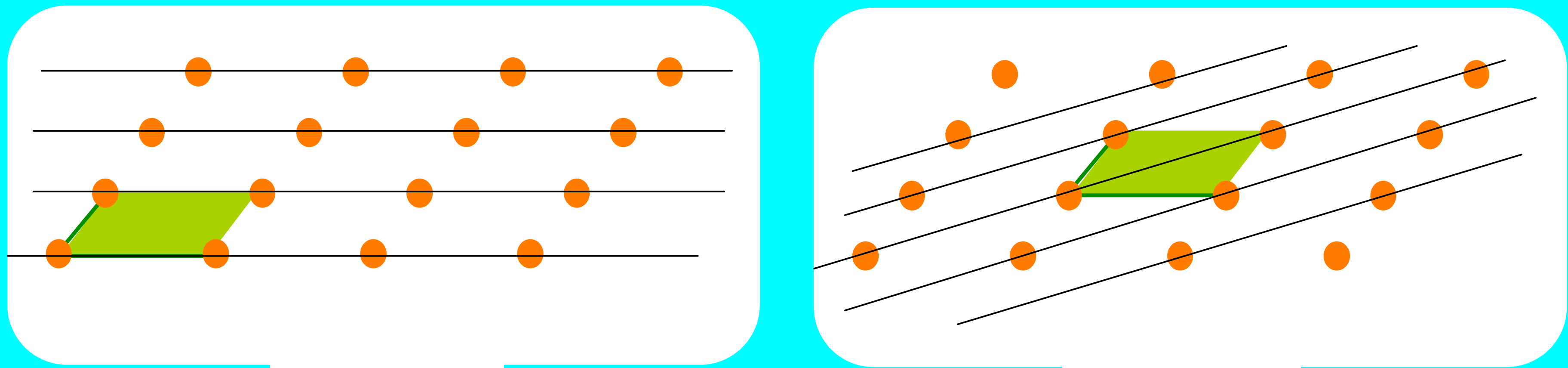
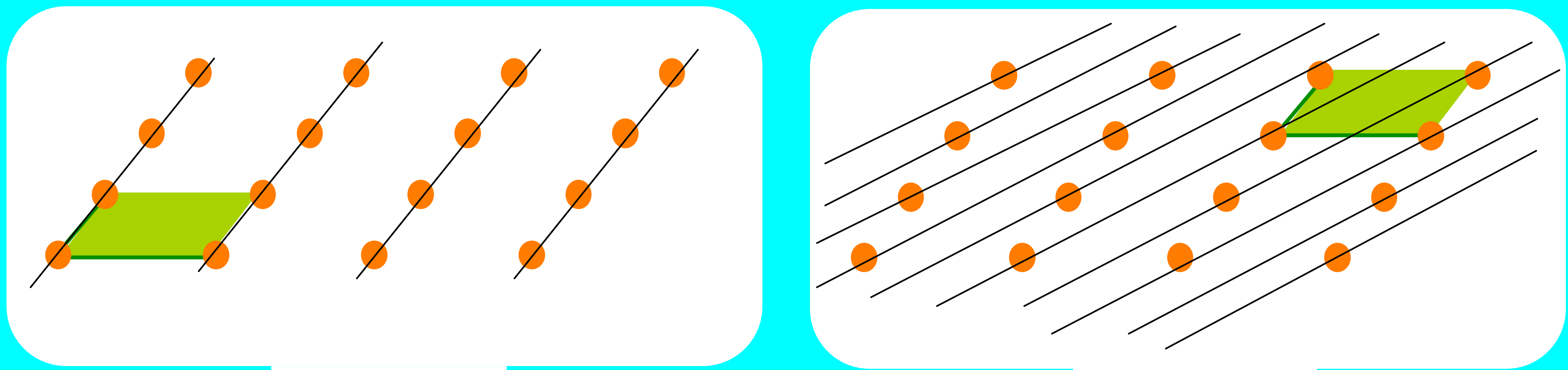


$(hk) = (01)$

$(hk) = (1\bar{1})$



$(hk) = (10)$

$(hk) = (2\bar{1})$

Miller indices, 3-D

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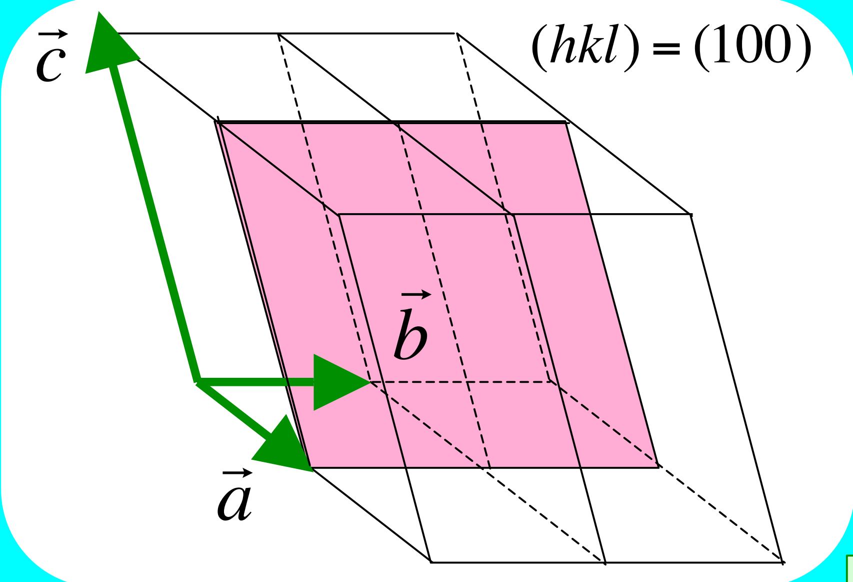
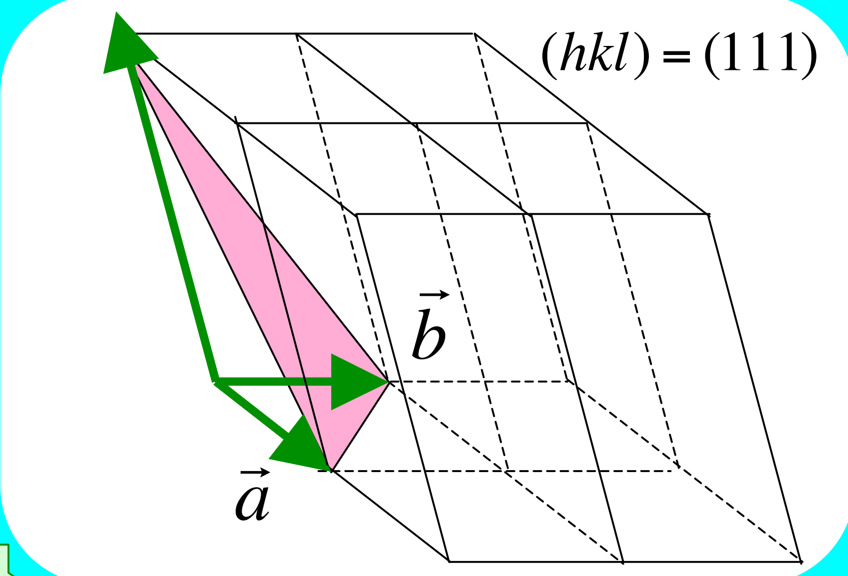
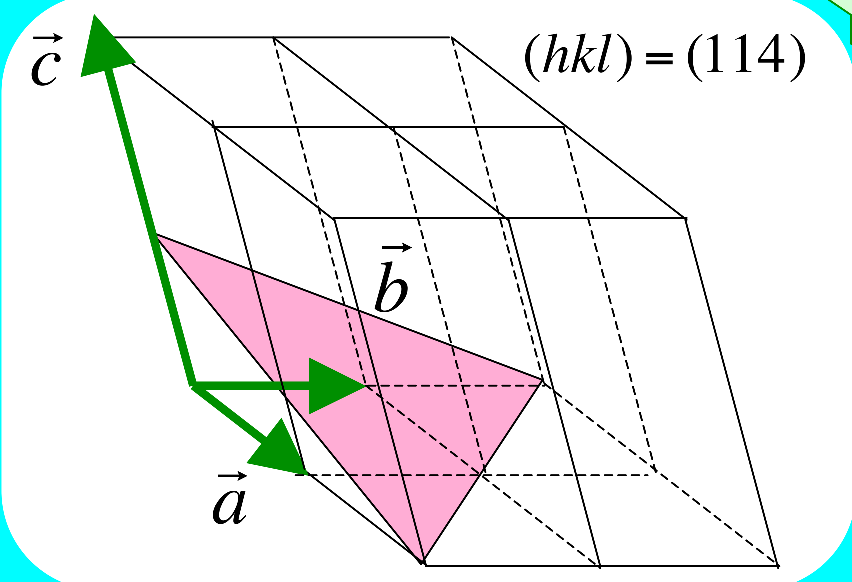
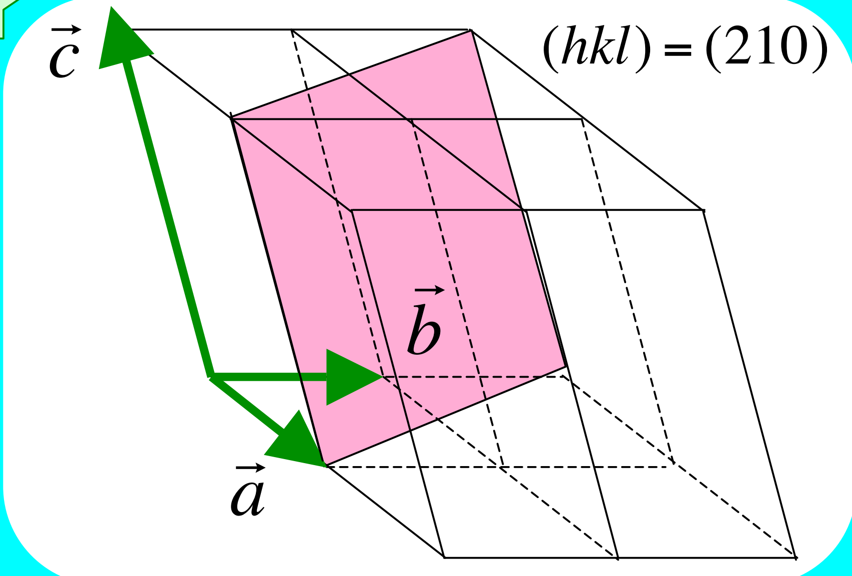




3-D





Miller indices, cubic lattices

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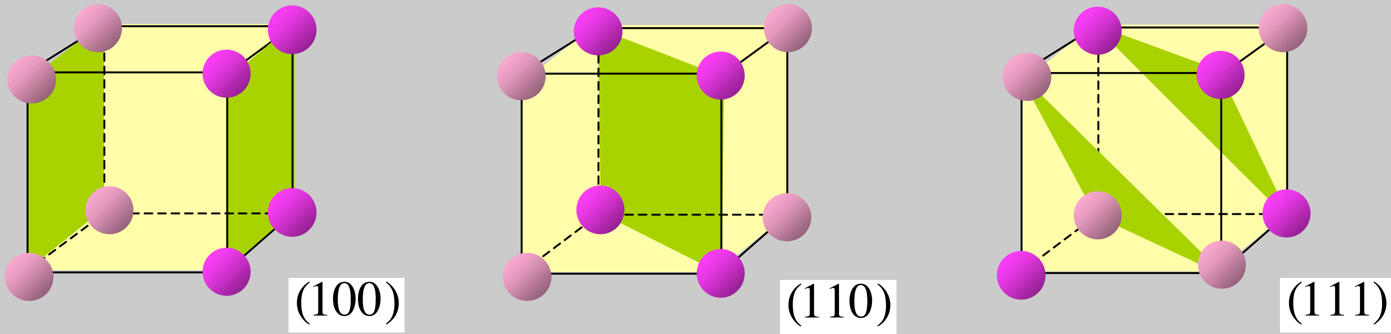
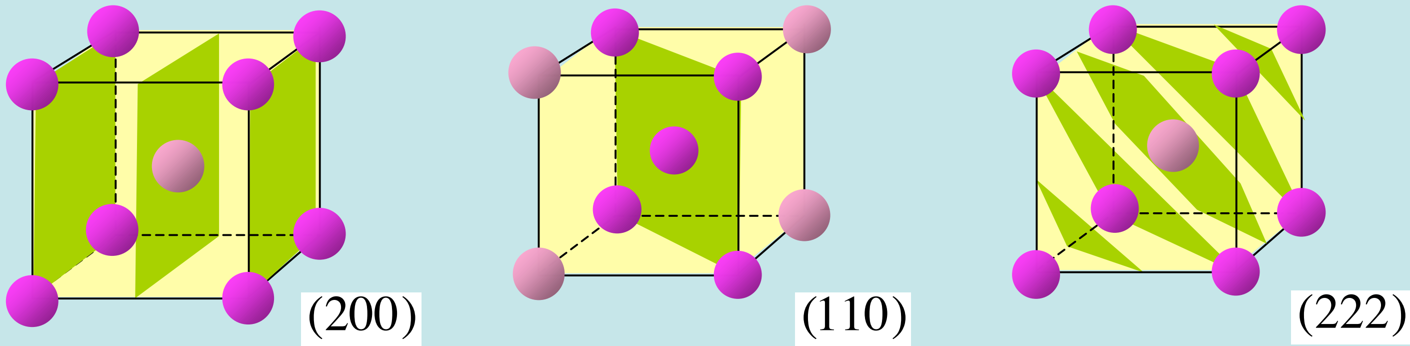
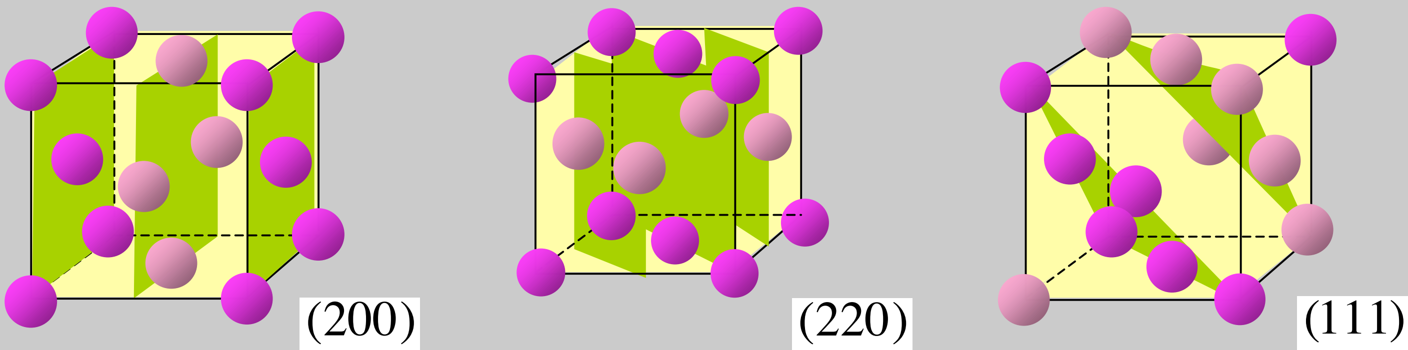
sc



bcc



fcc



Interplanar distance

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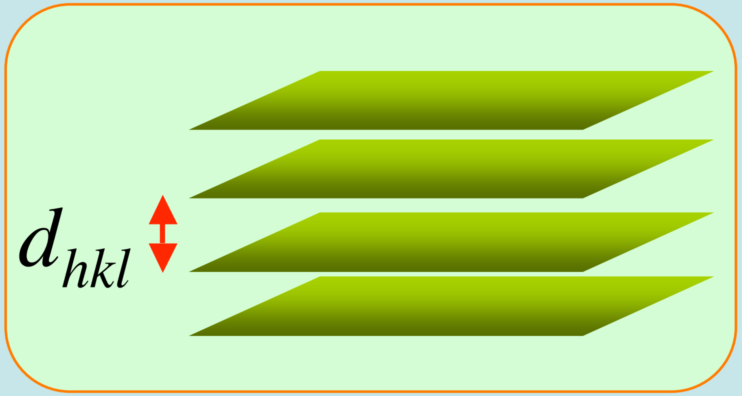
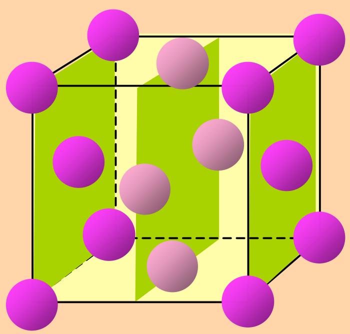


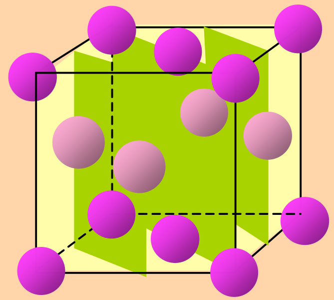
Cubic lattices

$$d_{hkl}^2 = \frac{a^2}{h^2 + k^2 + l^2}$$

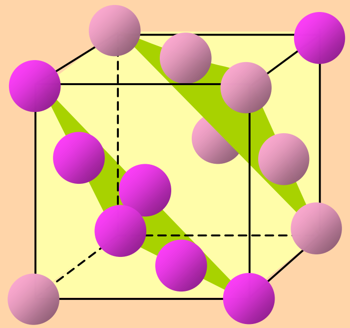
Copper, fcc, a=3.61 Å



$$d_{200} = \frac{a}{2} = 1.805 \text{ Å}$$



$$d_{220} = \frac{a}{2\sqrt{2}} = 1.276 \text{ Å}$$



$$d_{111} = \frac{a}{\sqrt{3}} = 2.084 \text{ Å}$$

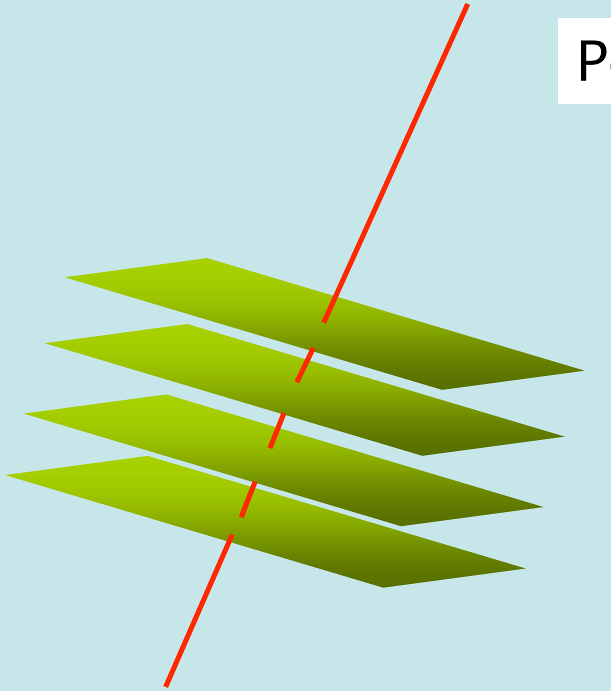
Planes and directions

Perpendicular direction

$[hkl]$

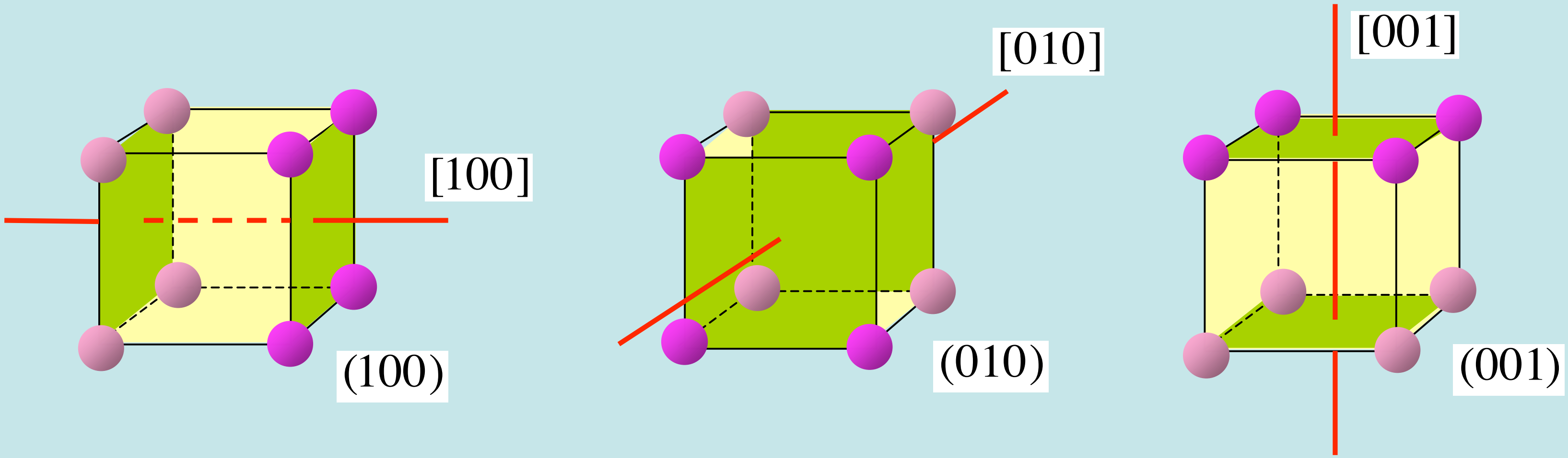
Family of planes

(hkl)



Equivalent planes and directions

Equivalent directions $<100>$



Equivalent planes $\{100\}$

Reciprocal lattice

X-ray diffraction pattern

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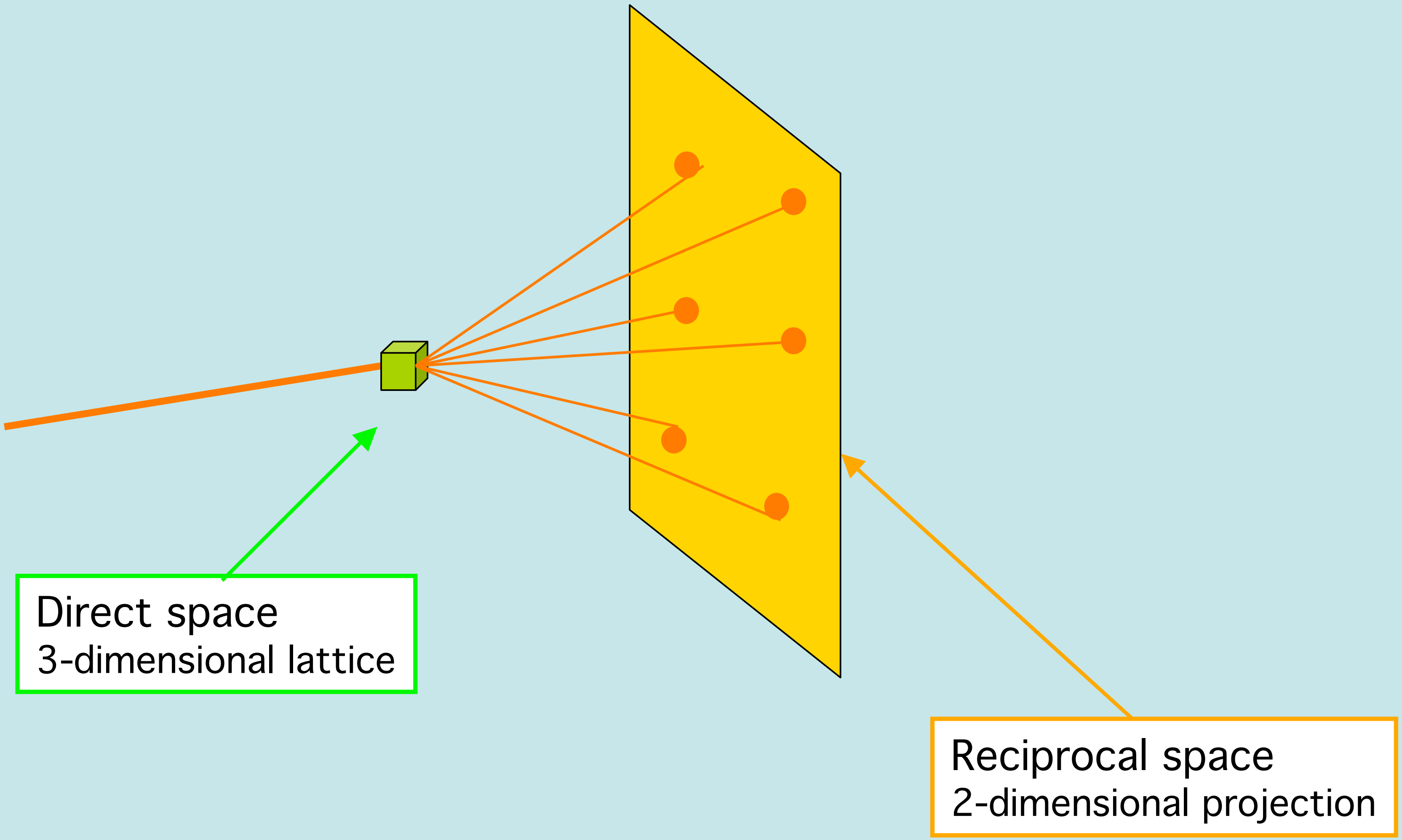

Basic idea

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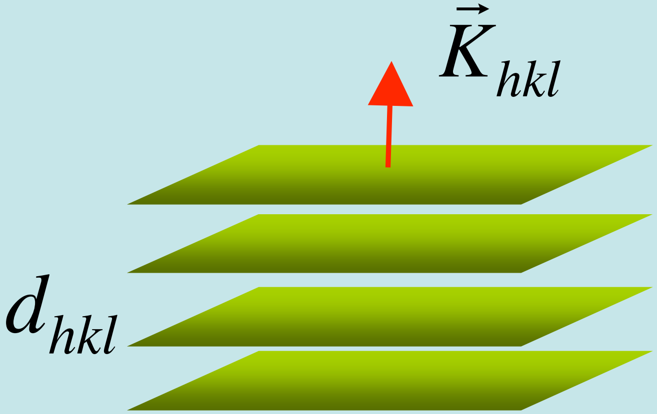
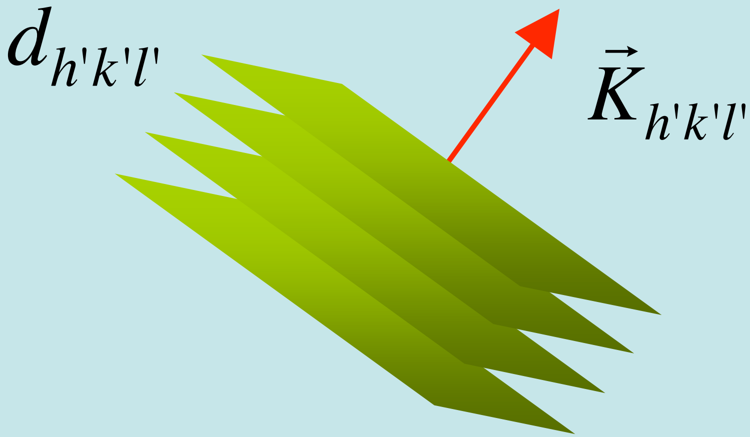
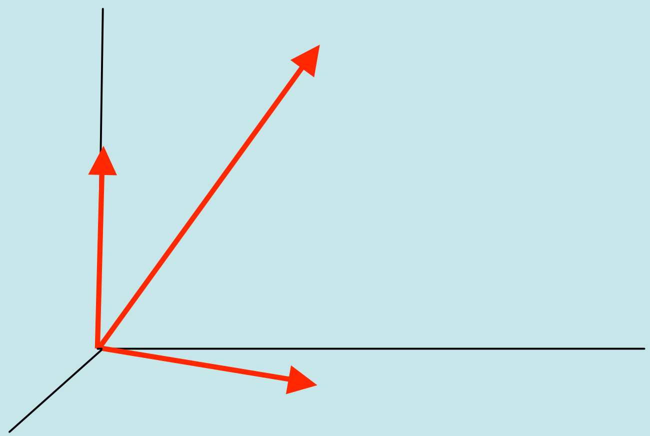
A) Family of planes → wave-vector



$$K_{hkl} = \frac{2\pi}{d_{hkl}}$$



B) Wave-vectors → set of points



C) Set of points → lattice

Reciprocal quantities

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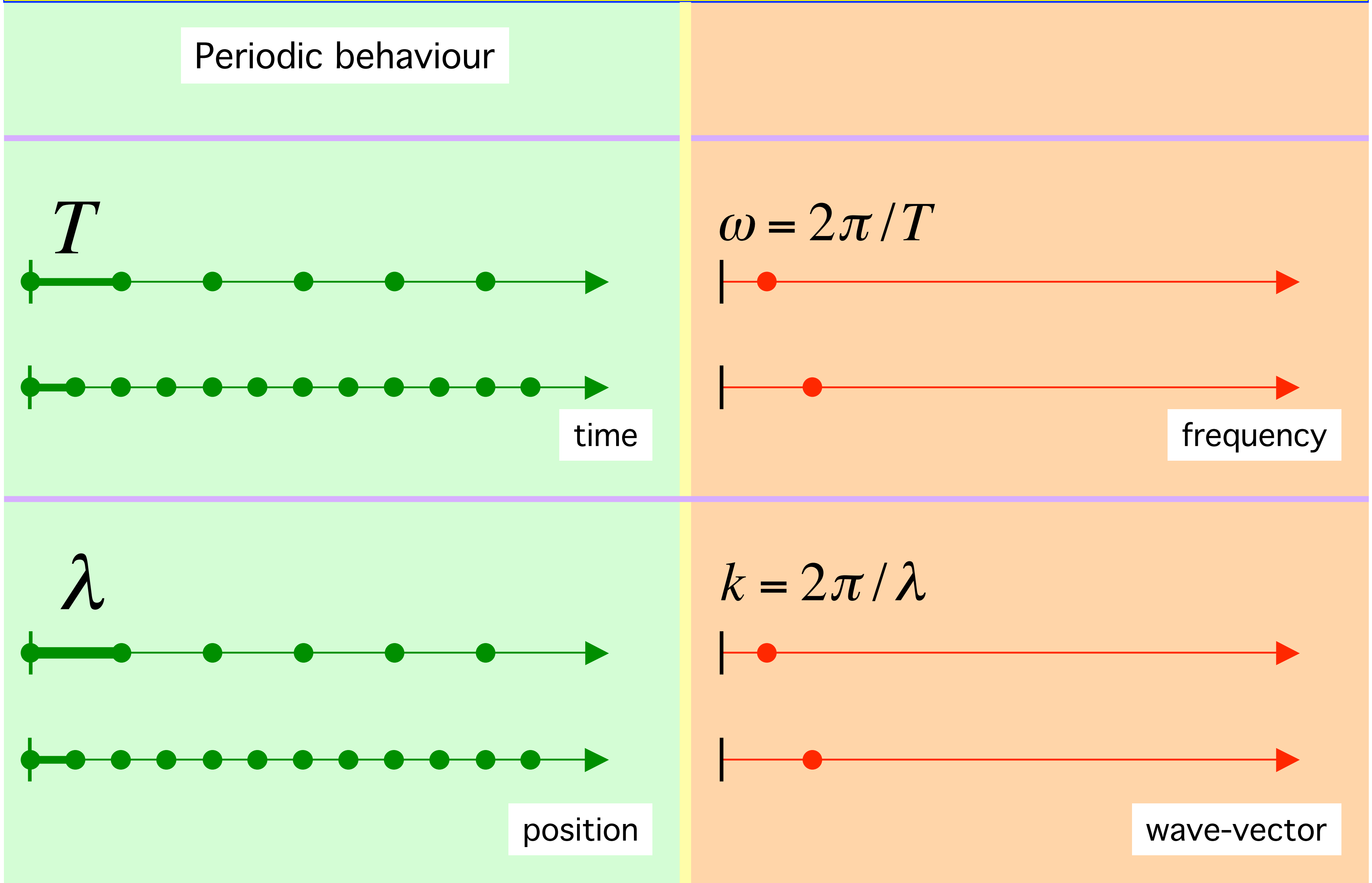

Harmonics

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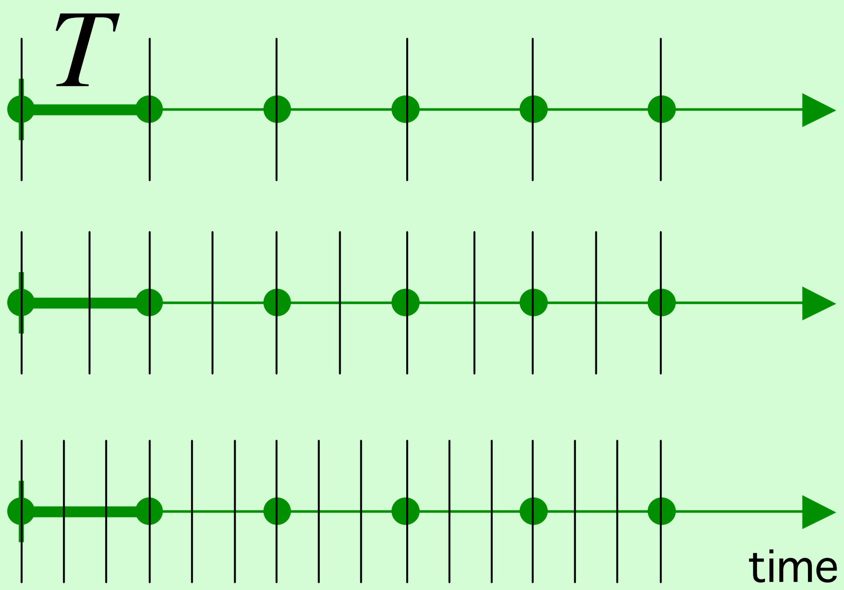
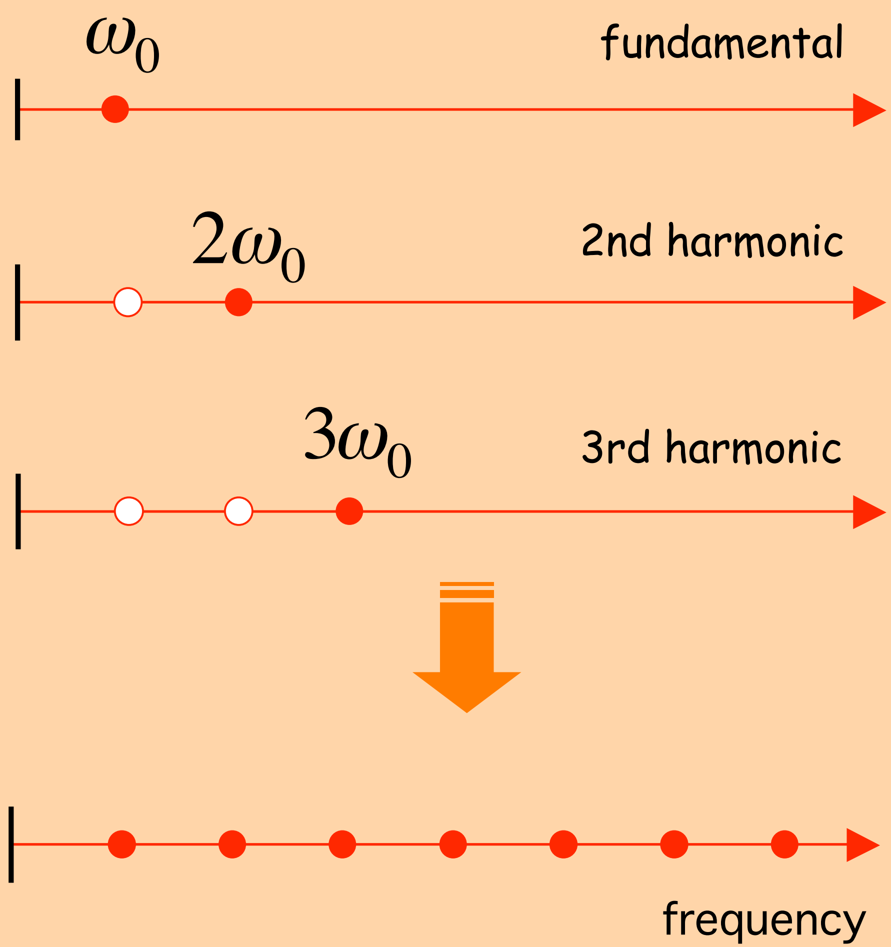
Periodic behaviour





Periodic behaviour

1-D

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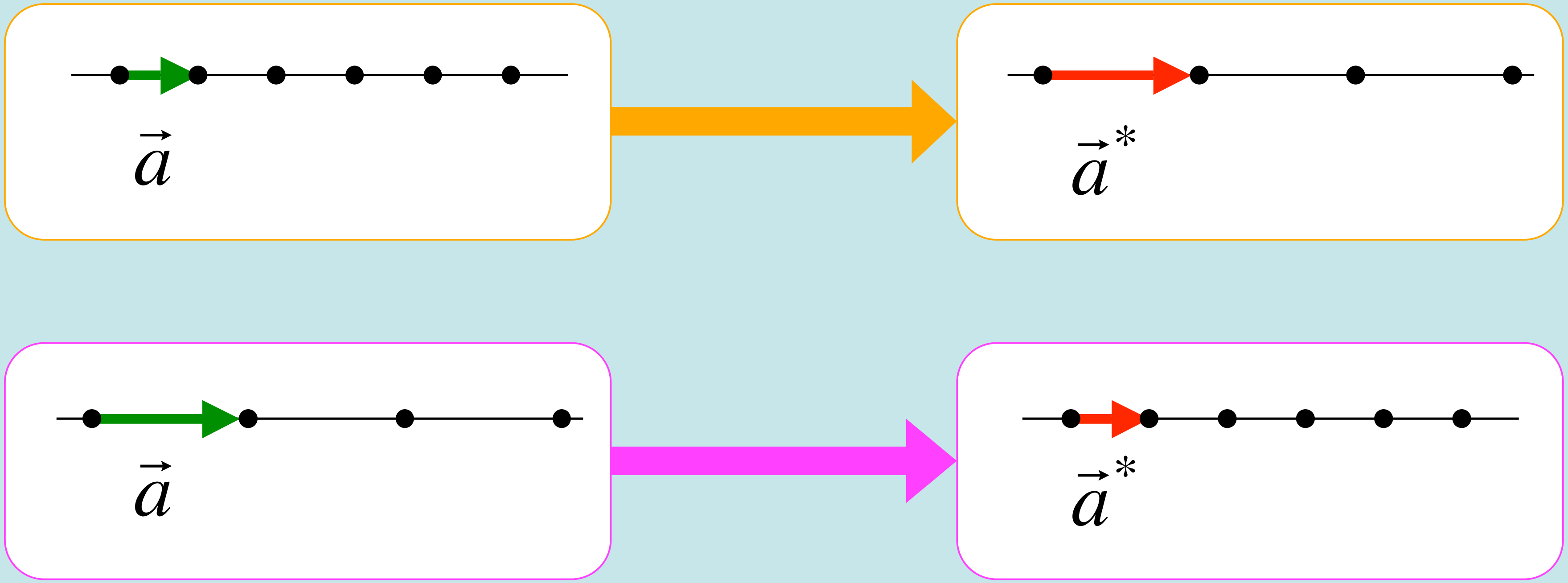
Direct space

Reciprocal space

$$a^* = \frac{2\pi}{a}$$



2-D, rectangular lattice (a)

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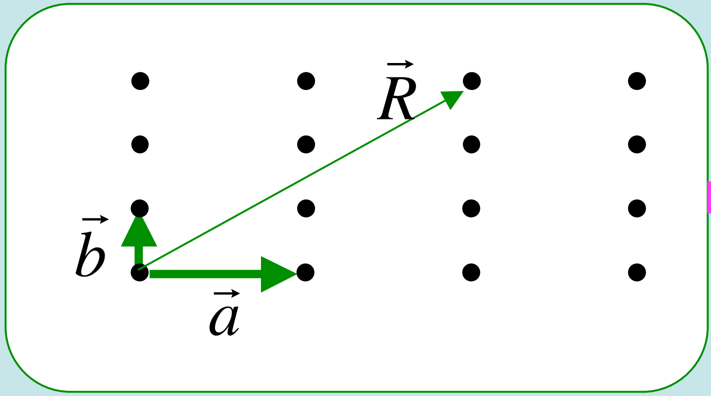
Direct space



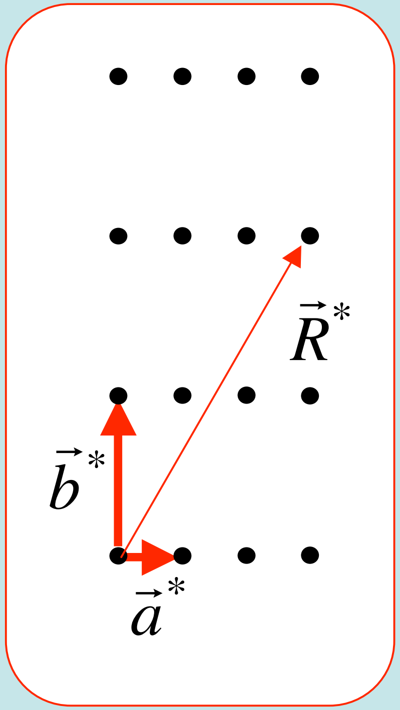
$$a^* = \frac{2\pi}{a} = \frac{2\pi b}{ab}$$

$$b^* = \frac{2\pi}{b} = \frac{2\pi a}{ab}$$

$$\vec{a}^* \perp \vec{b}$$

$$\vec{b}^* \perp \vec{a}$$

Reciprocal space



$$\vec{R} = n_1\vec{a} + n_2\vec{b}$$

$$\vec{R}^* = m_1\vec{a}^* + m_2\vec{b}^*$$

2-D, rectangular lattice (b)

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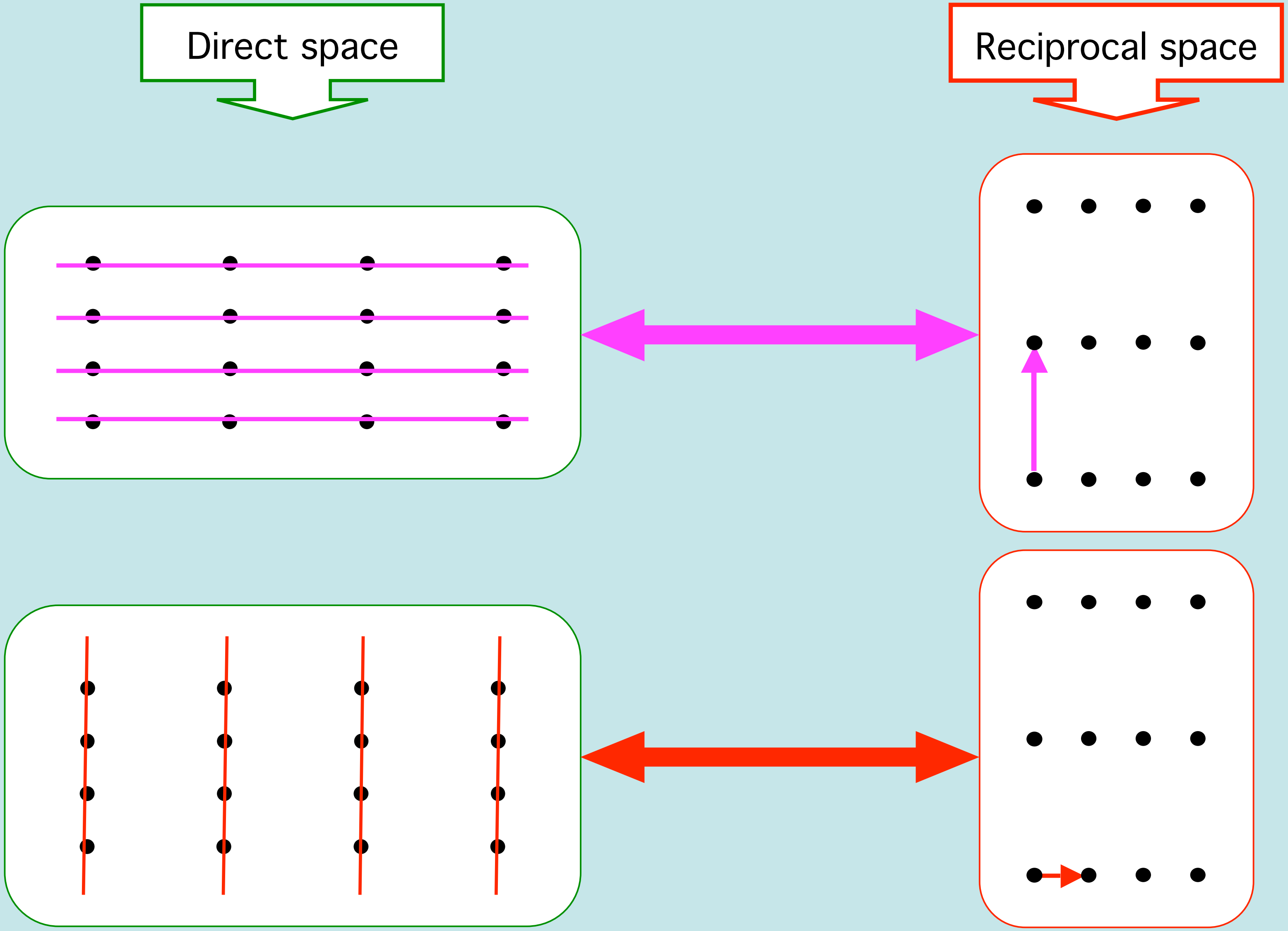

2-D, rectangular lattice (c)

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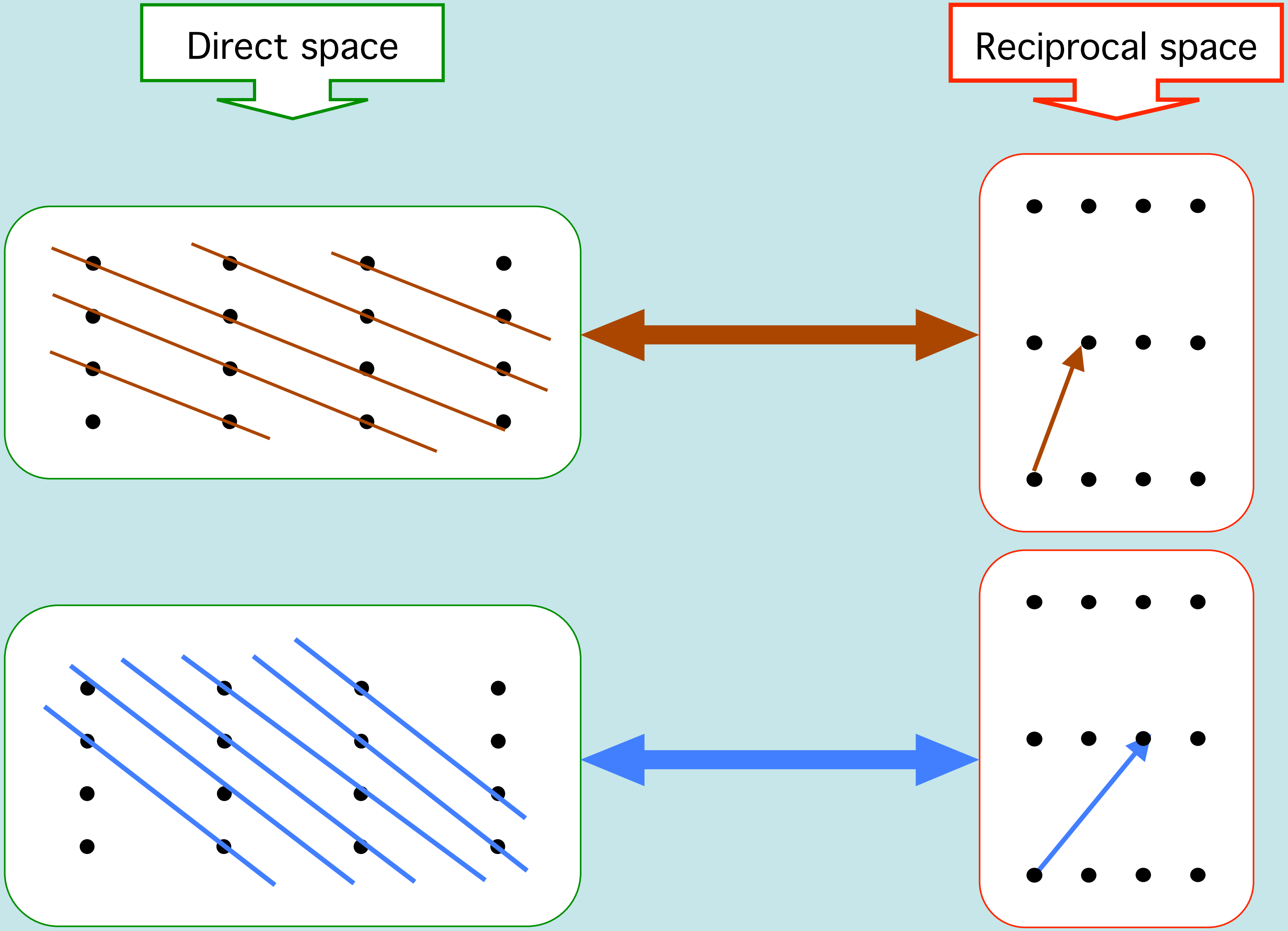

2-D, rectangular lattice (d)

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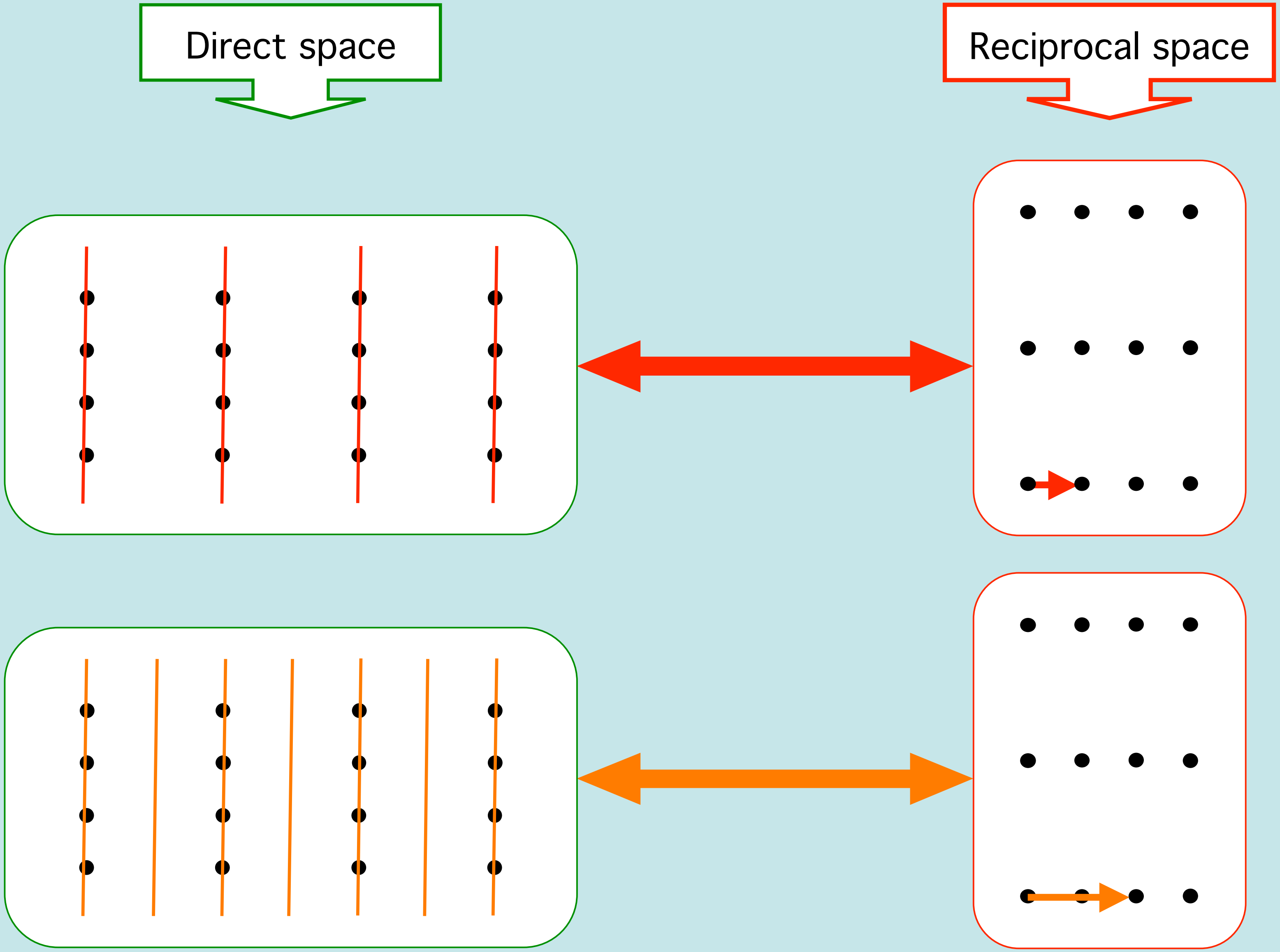

2-D, oblique lattice (a)

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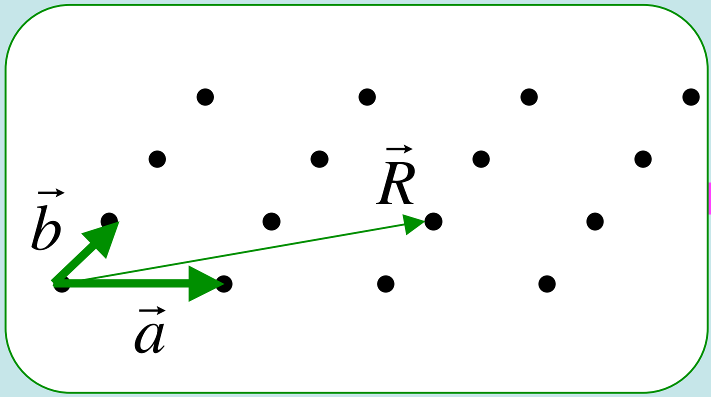
Direct space



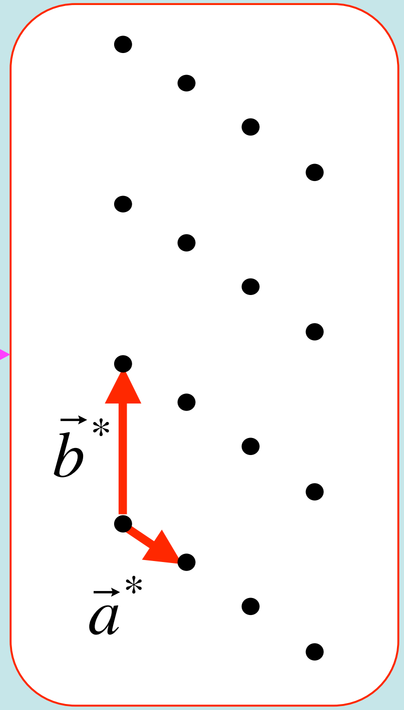
$$a^* = \frac{2\pi}{a\sin\theta} = \frac{2\pi b}{ab\sin\theta}$$

$$b^* = \frac{2\pi}{b\sin\theta} = \frac{2\pi a}{ab\sin\theta}$$

$$\vec{a}^* \perp \vec{b}$$

$$\vec{b}^* \perp \vec{a}$$

Reciprocal space



2-D, oblique lattice (b)

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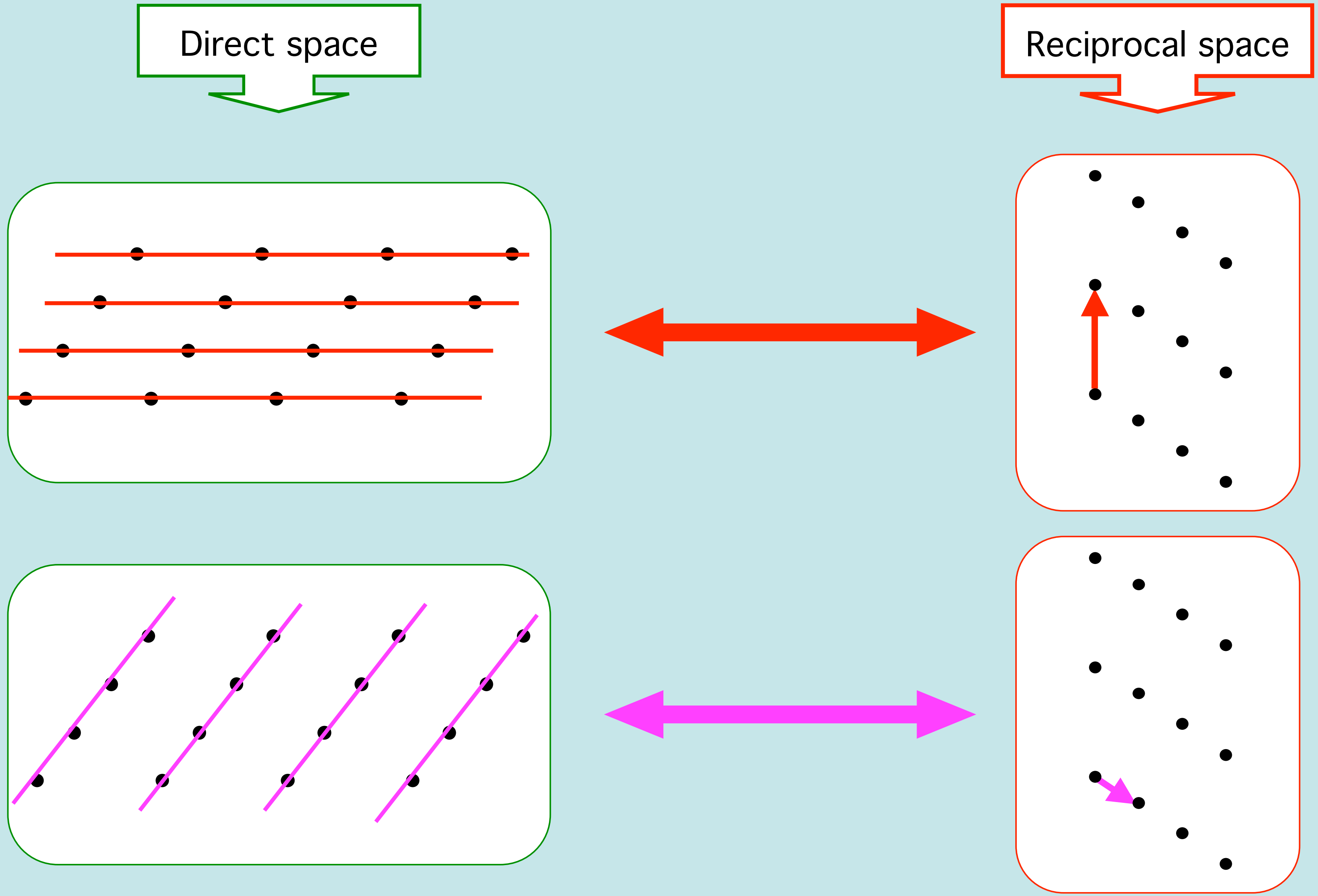

Cubic lattices (a)

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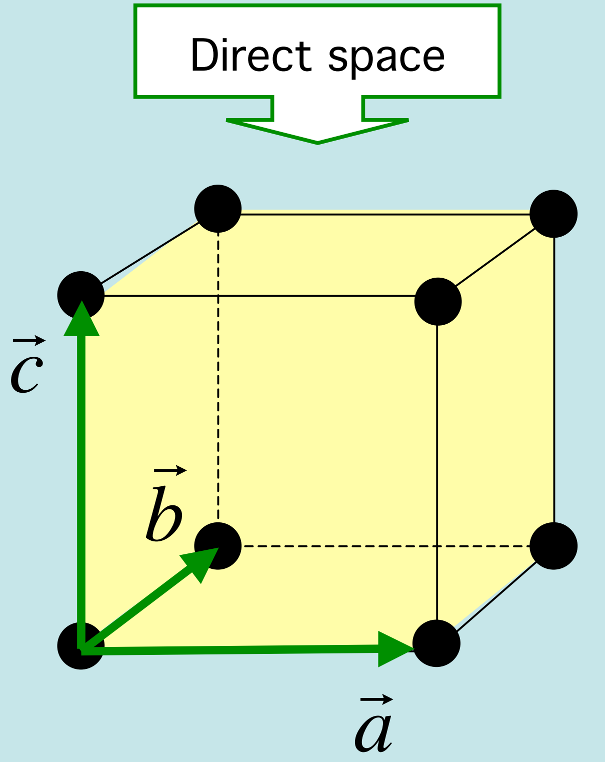

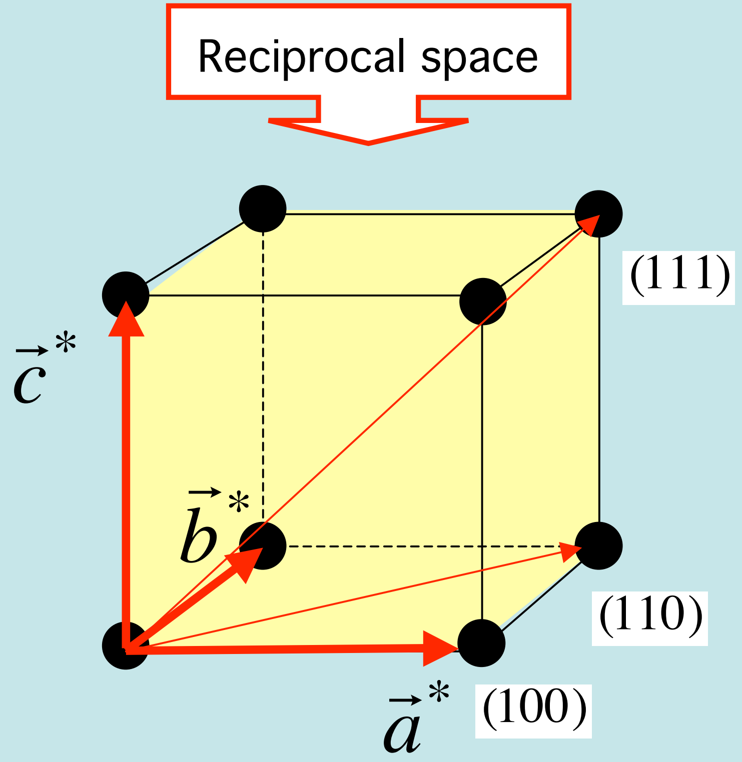

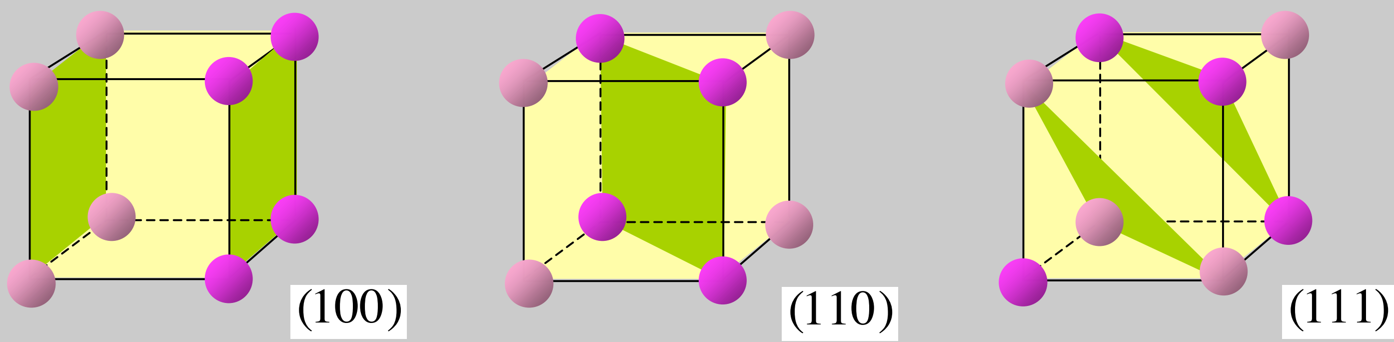

Cubic lattices (b)

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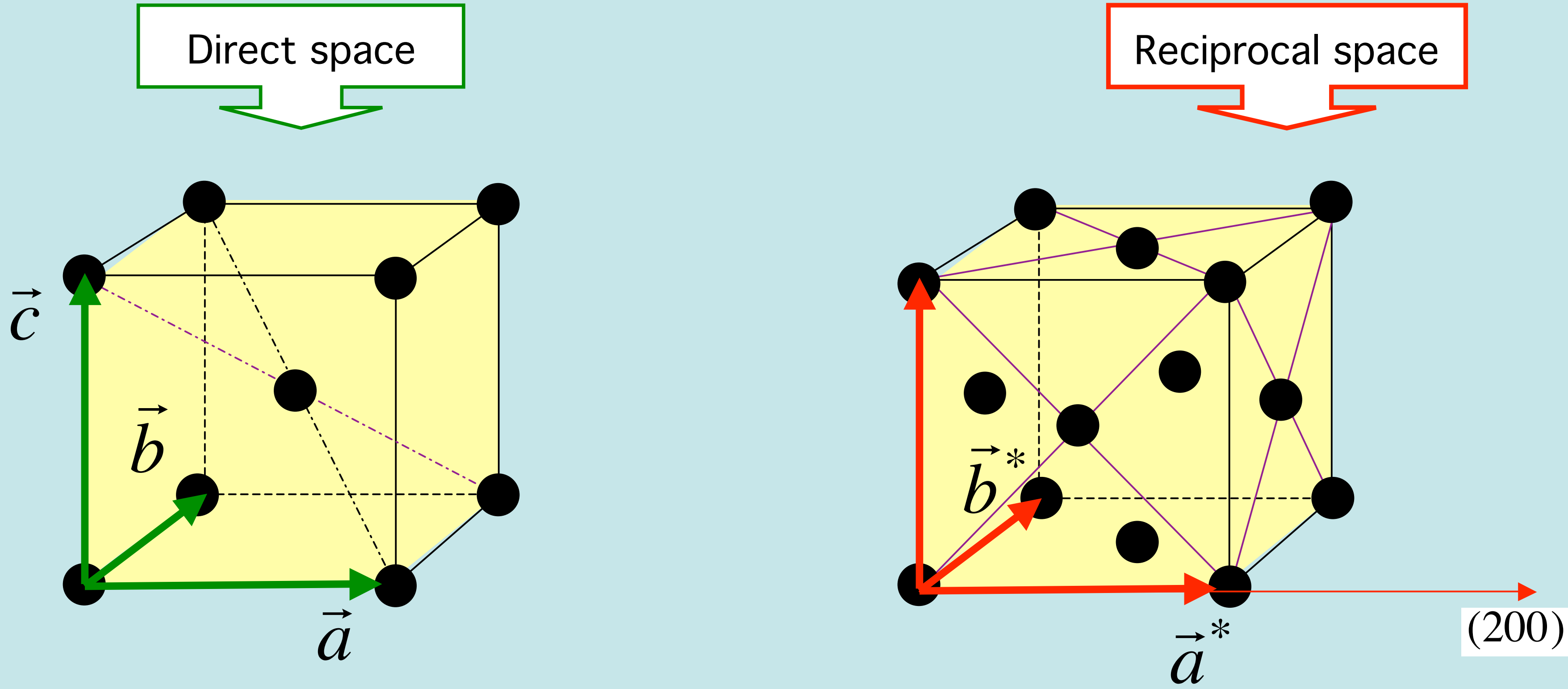

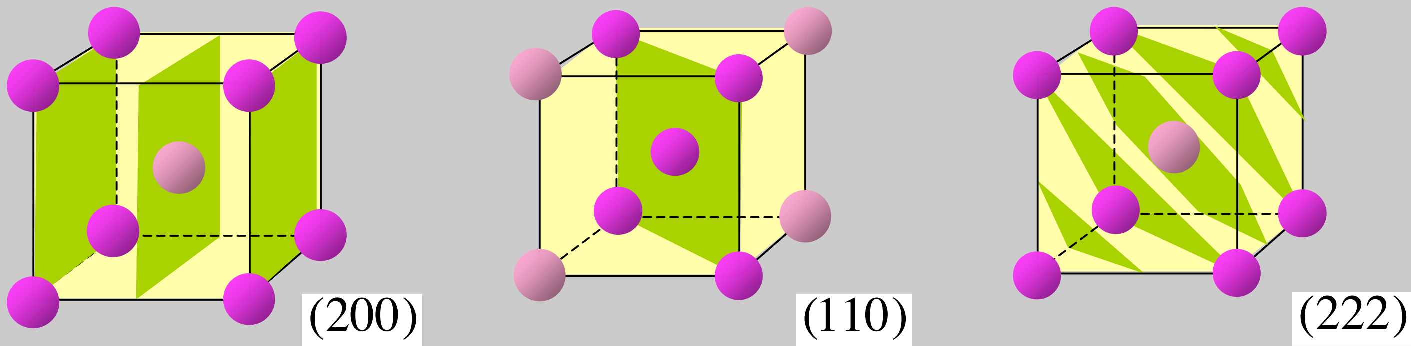

Cubic lattices (c)

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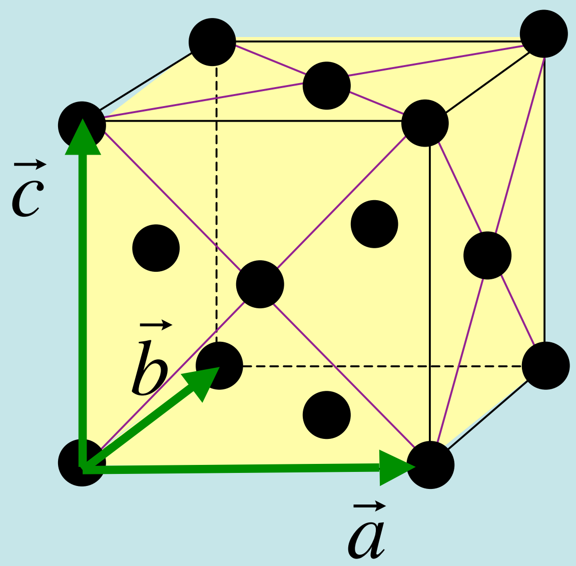
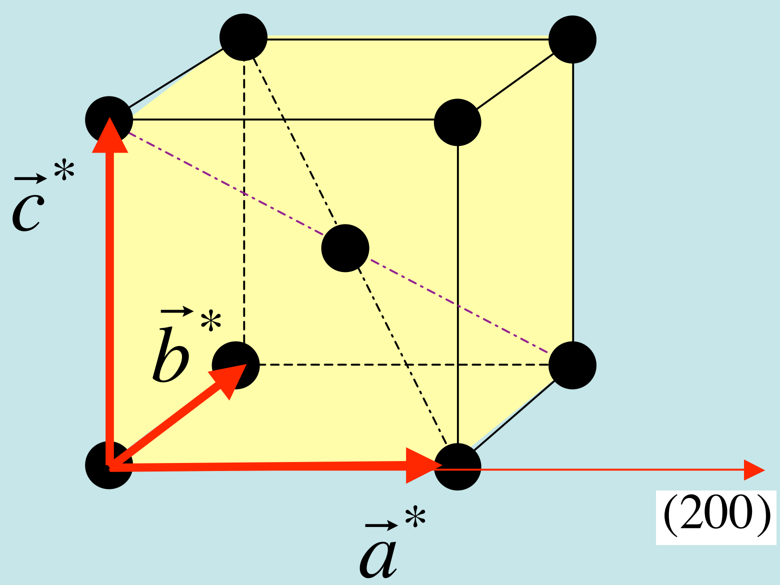
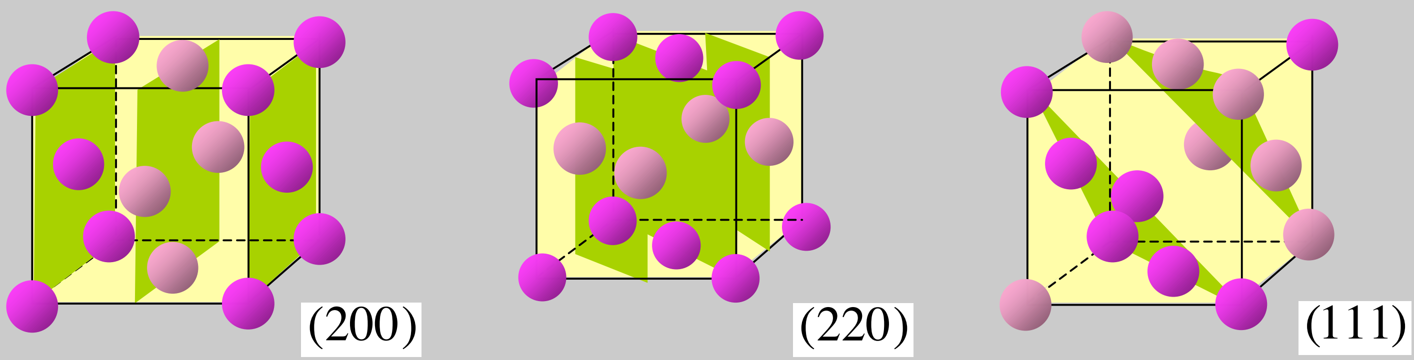
Direct space



Reciprocal space





Primitive vectors: general rule

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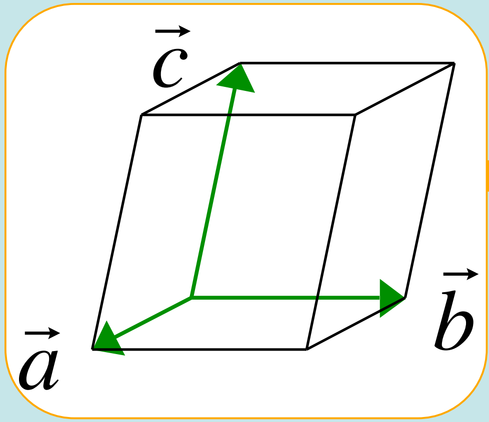
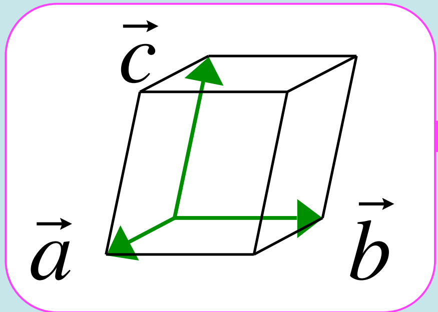
Direct space





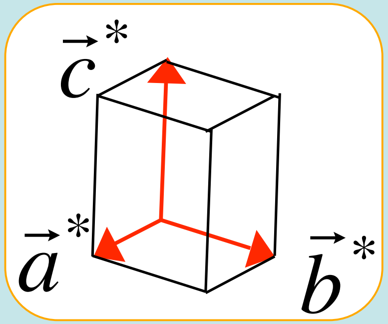
$$\vec{a}^* = 2\pi \frac{\vec{b}\times\vec{c}}{\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)}$$

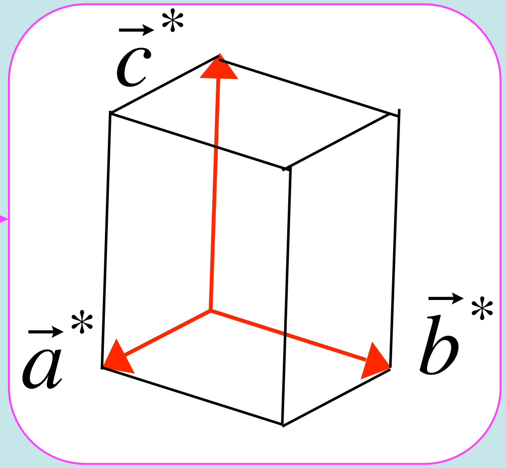
$$\vec{b}^* = 2\pi \frac{\vec{c}\times\vec{a}}{\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)}$$

$$\vec{c}^* = 2\pi \frac{\vec{a}\times\vec{b}}{\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)}$$

$$\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$$

Reciprocal space





Reciprocal lattice and lattice planes

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For any family of lattice planes separated by a distance d
there are reciprocal lattice vectors perpendicular to the planes,
the shortest of which have a length $2\pi/d$.

For any reciprocal lattice vector **R***,
there is a family of lattice planes normal to **R***
and separated by a distance d,
where $2\pi/d$ is the length of the shortest reciprocal lattice vector
parallel to **R***.

- Microscopic structure of materials

Macro and micro-crystals

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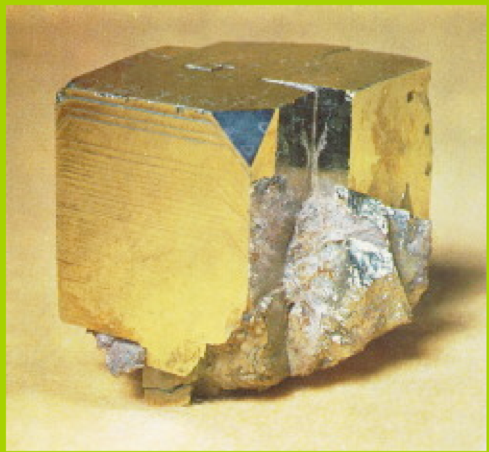
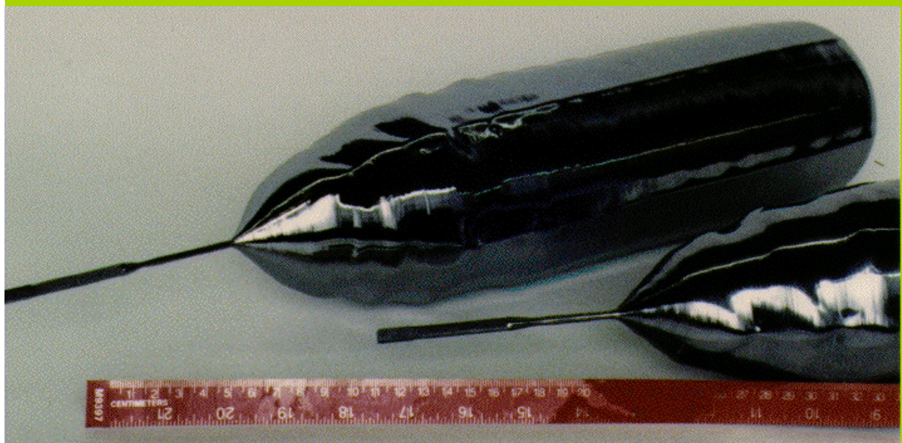
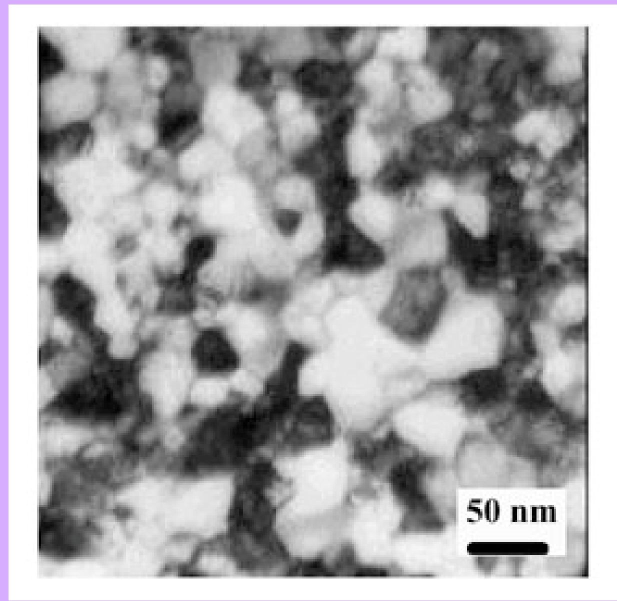
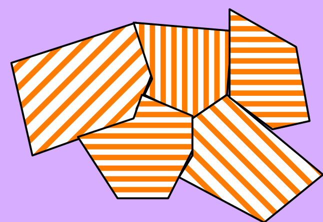


Monocrystalline silicon, ∅ 13 cm





Cr, electron microscopy



Grain structure

Effects of temperature

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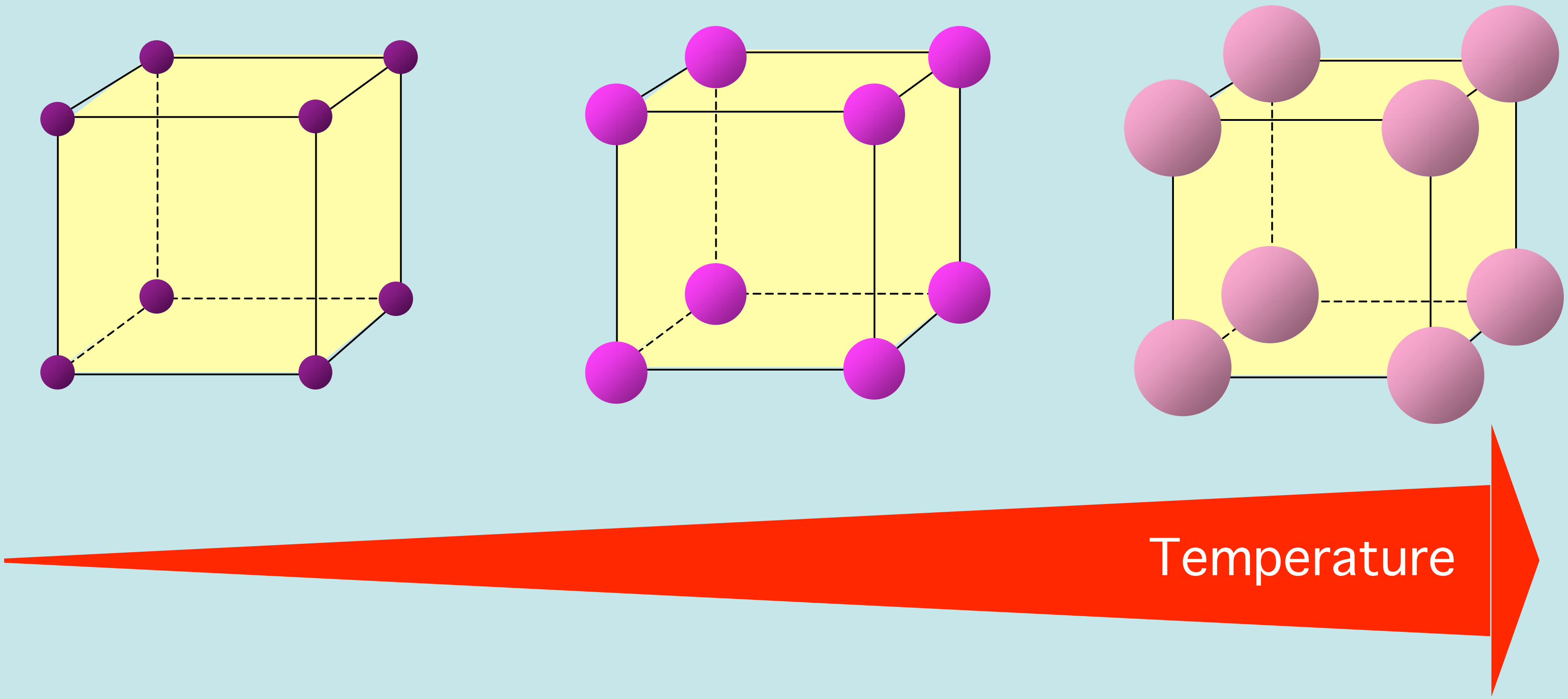

Thermal motion → Spread of atomic positions

Crystalline and non-crystalline materials

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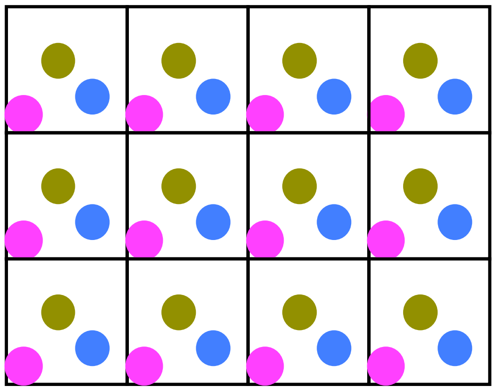
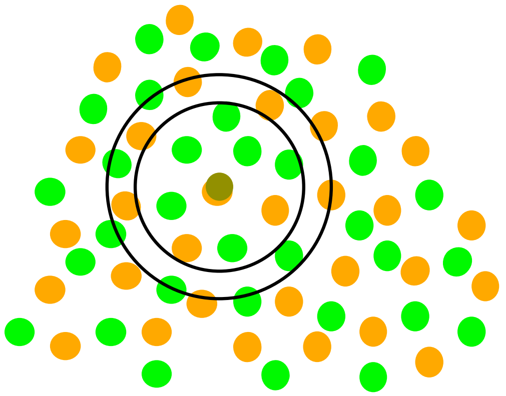
Crystalline solids



Long-range order

Non-crystalline systems



No long-range order

Cooling rate

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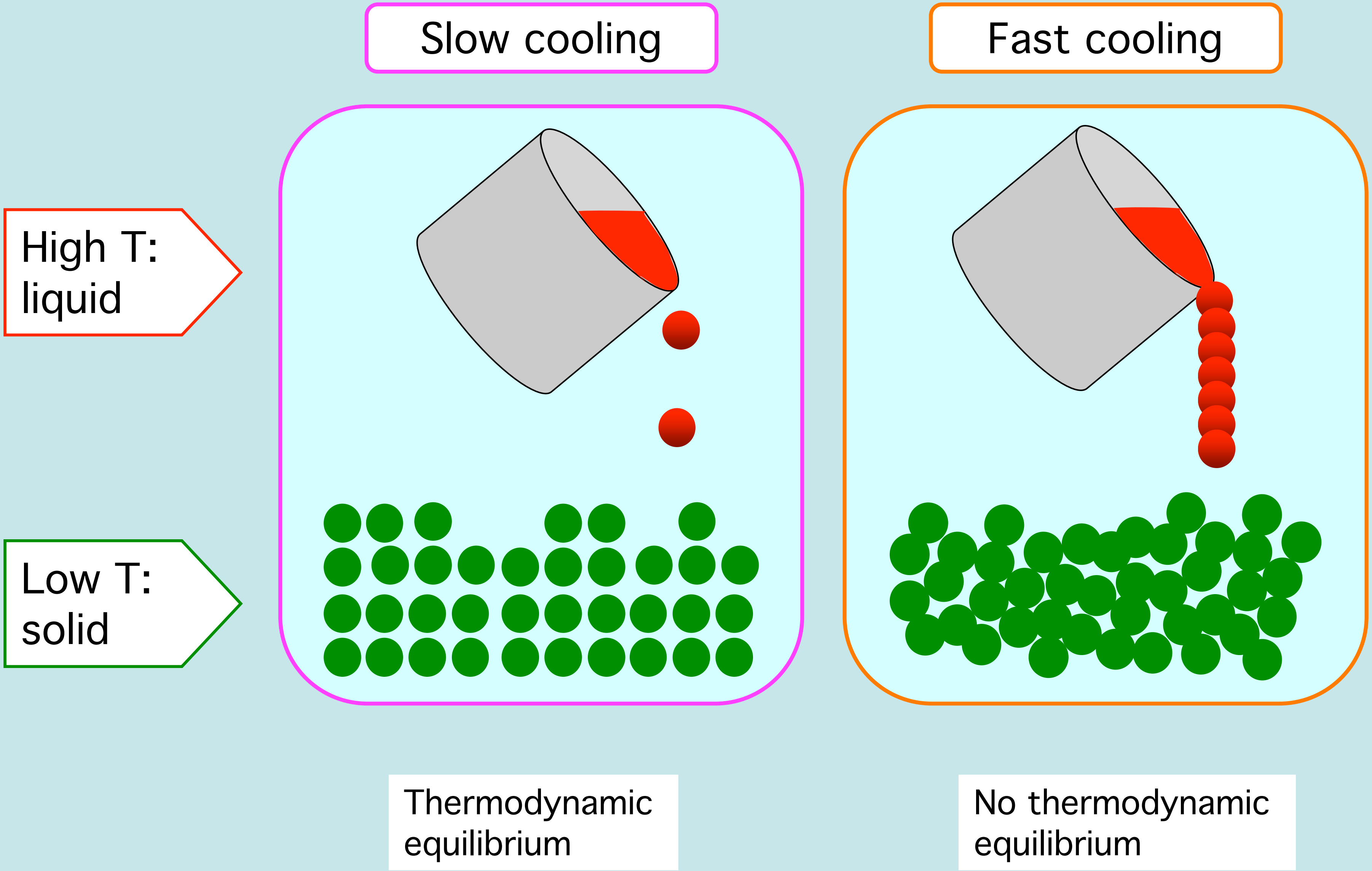

Radial Distribution Function

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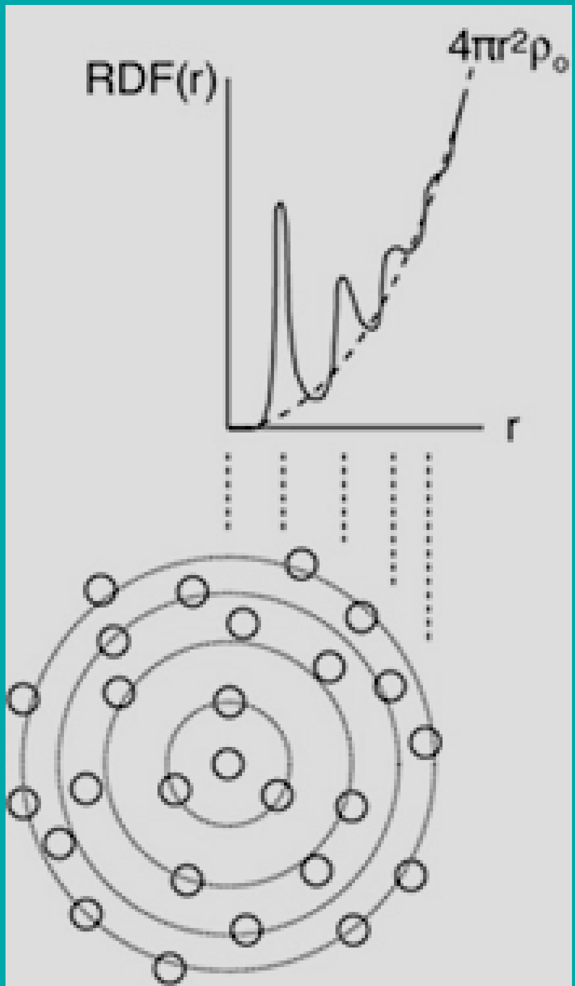
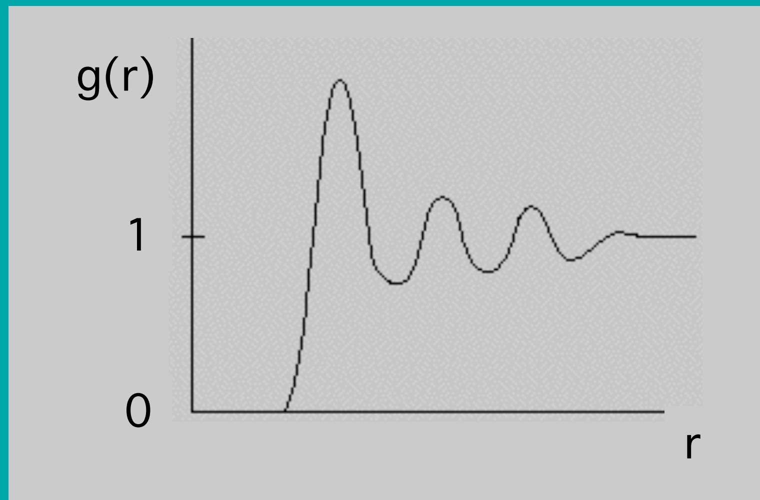


average density

$$RDF = 4\pi r^2 \rho(r) = 4\pi r^2 \rho_0\, g(r)$$

PDF = Pair Distribution Function



Short-range order

Summary

- Plane waves and wavevector
- Crystal structure = Bravais lattice + basis
- Bravais lattices: primitive vectors, unit cells (primitive and conventional), classifications
- Crystal structures (sc, bcc, fcc, hcp ...)
- Crystal planes and Miller indices
- Reciprocal lattice
- Crystalline and non-crystalline materials


Dolomite mountains in winter